Railway Curves

An IRICEN Publication
RAILWAY CURVES

March 2010
(Corrected & Reprinted : November 2018)

INDIAN RAILWAYS INSTITUTE OF CIVIL ENGINEERING
PUNE - 411 001
Foreword to the corrected and updated version

The book on Railway Curves was originally published in March 2010 by Shri V B Sood, the then professor, IRICEN and reprinted in September 2013.

The book has been again now corrected and updated as per latest correction slips on various provisions of IRPWM and IRTMM by Shri V B Sood, Chief General Manager (Civil) IRSDC, Delhi, Shri R K Bajpai, Sr Professor, Track-2, and Shri Anil Choudhary, Sr Professor, Track, IRICEN.

I hope that the book will be found useful by the field engineers involved in laying and maintenance of curves.

Pune
November 2018

Ajay Goyal
Director
IRICEN, Pune
PREFACE

In an attempt to reach out to all the railway engineers including supervisors, IRICEN has been endeavouring to bring out technical books and monograms. This book “Railway Curves” is an attempt in that direction. The earlier two books on this subject, viz. “Speed on Curves” and “Improving Running on Curves” were very well received and several editions of the same have been published. The “Railway Curves” compiles updated material of the above two publications and additional new topics on Setting out of Curves, Computer Program for Realignment of Curves, Curves with Obligatory Points and Turnouts on Curves, with several solved examples to make the book much more useful to the field and design engineer.

It is hoped that all the P.way men will find this book a useful source of design, laying out, maintenance, upgradation of the railway curves and tackling various problems of general and specific nature.

Pune, Dated 29-02-2010

A. K. GOEL
Director
Indian Railways Institute of Civil Engineering
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FOREWORD

I joined IRICEN in April 2007. My designation at that time was Professor Track-2 and I was asked as to which track subject I will take up for teaching. I opted for "Curves" considering that this is one of the easier topics in Permanent Way. When I was asked whether I will write a book on curves, I was more than willing even though I had scarcely started teaching the subject at the time. During the writing of this book, I discovered the beauty and complications in the simple subject of curves and their realignment.

This book has been written for the field engineer whether in construction or maintenance of railway curves. It is not meant as text book reference and hence is not very heavy in theory. But it does have enough theory to explain the basis behind many of the provisions in the manuals. The focus, however, is to provide the reader with provisions in the manuals as well as maintenance tips to avoid problems in laying/ maintaining curved tracks. In this book, I have given references to the manual paras wherever relevant. This will help reader's cross verify the facts and read the actual manual provisions so that there is no ambiguity regarding actual action to be taken. This will also help when the manual provisions change. I have tried to cover the special locations such as bridges, level crossings etc in curves as these take up lot of time and efforts of the maintenance engineers.

This book is not really an original work. This book owes its contents to a lot of people – all the IRICEN faculty who have been teaching the subject in the past and who have compiled wonderful notes on the subject in the library, all the people who have participated in numerous discussions on the IRICEN Discussion forum and who have helped me get lots of answers, all the guest officers at IRICEN who have been very inquisitive, very knowledgeable and have enabled me to put together this book, paragraph by paragraph, all the people whom I have known and have pestered to get the field experience on various aspects of curve laying, inspection, maintenance and realignment. This
leaves space for me only as a compiler of the information. This book owes major part of the contents and examples to the two books on “Speed on Curves” and “MAN ON THE JOB- Improving Running on Curves” published previously by IRICEN. These two books were very popular and have been reprinted several times in the past. Since the books were essentially dealing with one subject only and the readers had to move from one book to the other to get the information desired, I decided to merge the two and publish a single book which is now in front of you.

I will name a few people whose work directly finds place in the book: Shri M S Ekbote, Retired AMCE, Rly Bd whose computer program we have been using for realignment of curves and the instructions for use of the program are in Annexure I, and Shri Manoj Arora, Professor (Track Machines), IRICEN who has written the parts of the book regarding laying of points and crossings sleepers on curve and tamping of curves. Smt Gayatri Nayak, my CA and Shri Sunil Pophale also require to be thanked for the efforts put in by them in writing the book.

Above all, thanks are due to our Director, Shri A. K. Goel, for his constant encouragement for out of box thinking which has enriched this book and greatly enhanced its value. Without the same, this book would not have come out in the present form.

“To err is human and to point it out is reader’s duty” is what I will like to say. Despite ample care taken in compiling the book, some errors are quite likely to have crept in. I apologize for the same and request the readers to send their suggestions to IRICEN at mail@iricen.gov.in so that these can be kept in mind whenever the next reprint/ version are to be prepared.

Pune
January 2010
V B Sood
Professor
IRICEN, Pune
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COMMONLY USED TERMS IN THE BOOK

BG  Broad Gauge track, 1676 mm gauge
BG SOD  Indian Railways Schedule of Dimensions
         1676 mm Gauge, Revised 2004
C_a  Actual Cant or superelevation provided
C_d  Cant Deficiency
C_ex  Cant Excess
C_a_max  Maximum actual Cant or superelevation permissible
C_d_max  Maximum Cant Deficiency permissible
C_ex_max  Maximum Cant Excess permissible
cm  Length or distance in centimetres
D  Degree of curve
Equivalent circular curve  A conceptual circular curve without considering any transitions. The transitions are added later on at either end to get the actual proposed curve.
G  Dynamic Gauge or center to center of the running rails, 1750 mm for BG and 1080 mm for MG
g  Acceleration due to gravity, 9.81 m/sec^2
IR  Indian Railways
IRPWM  Indian Railways Permanent Way Manual second reprint 2004
IRTMM  Indian railways Track Machines Manual, March 2000
KMPH  Speed in Kilometers Per Hour
L  Length of transition; also, extra clearance due to Lean of train in Chapter II
m  Length or distance in metres
MG  Meter Gauge track, 1000 mm gauge
mm  Length or distance in millimetres
m/sec  Speed in metres per second
m/sec² Acceleration in metre per second square
NG Narrow Gauge track, 762 mm or 610 mm gauge
Offloading A condition where the load on a wheel is less than half the nominal axle load.
Onloading A condition where the load on a wheel is more than half the nominal axle load.
PWI Permanent Way Inspector, Refers to Senior Section Engineer, Section Engineer or Junior Engineer looking after the Permanent Way or Track on Indian railways. The term may also include the Permanent Way Supervisor/ Gang Mate etc who might look after the maintenance work in the track.
S Extra clearance for sway in vehicle on curves
Slew Shifting the alignment of curve. It can be outwards or inwards and is generally expressed with a sign depending on the sign convention.
R Radius of curve
V Speed of train
V_{eq} Equilibrium Speed
V_g or V_G Booked speed of goods trains
V_{max} Maximum speed permissible on the curve
CHAPTER I

UNDERSTANDING CURVES
1.1 Introduction: Curves form an indispensable component of railway alignment. It is desirable, but not always possible, to follow the straight alignment. The alignment has to be changed by introduction of curve(s) due to various techno-economic reasons such as:

- The sources of traffic i.e. ports, industries, mines etc have to be connected
- The important towns/ cities near the alignment have to be connected
- If some area in straight alignment is already built up, or there are obstructions such as water bodies, hills, utility services etc, and the track can not continue in straight alignment
- Certain geological formations and faults etc are inherently unstable and if the track is laid on these, there will be problems in maintenance. Such formations are avoided by changing the alignment and track is laid on better geological formations.
- From technical and economic considerations, rivers shall be crossed at the locations where the river alignment is stable, and length of the bridge to be provided is minimum. The track alignment has to be changed to cross the river at the best possible location.
- While climbing steep hills, it is possible that the straight alignment has excessive gradients. In such cases, track length has to be increased by introducing curves so as to keep the gradients within desirable limits.

Due to the above and other similar reasons, approximately 16% of track is in curve on BG/MG on Indian Railways, and substantially higher percentage on NG is located on curves.
Nature of forces experienced by vehicle is different on curved track as compared to straight track. The forces and their effect on track, vehicle and passengers are required to be systematically studied so that curves are properly designed and easily maintained. Due to the interplay of the forces on a curved track, it has been estimated that the maintenance effort on curves is about 25% extra over that on a straight track. Therefore, understanding the vehicle movement on curves and proper laying, maintenance, realignment of curves etc is important. With better understanding of curves, track engineers will be in better position to properly manage the curves on the railway system. The curves have, therefore, been considered as necessary evil.

Note: The railway curves have the property that their radii are generally quite large and this property is used for making many approximations, which are valid, especially when seen in comparison with the least count of methods of measurement employed in field.

1.2 Identification of a curve: A railway curve is a track which changes alignment without any sharp corners in rails. This change of alignment through curves is best attained through a circular curve. A circular curve has the advantage of uniform curvature i.e. uniform change of direction which makes the task of management of forces during change of alignment easier. A circular curve used on railway system is identified with the following parameters:

- Radius, R or Degree, D
- Direction of Curve (LH or RH)

The radius R is the radius of the circle at the center line of the track, part of which is the curved alignment for the railway track. Degree of curve D is the angle subtended by an arc of 30.5m length at the centre of the same circle (Fig.1.1).

The circumference of the curve, 2πR, subtends an angle 360° at the center of the curve. Therefore, the angle subtended by 30.5 m chord (which is taken approximately equal to the 30.5 m arc as the radius of the curve is quite
large) at the center of curve can be worked out as follows:

The direction of curve is determined by the change in direction as seen in the direction of movement of trains. **Left hand (LH) curve** is there if the change in direction of the curve is in counterclockwise direction when seen in the direction of travel in multiple lines or in the direction of increasing kilometers in case of single line. Similarly, **Right**

Since arc length of 30.5m corresponds to $D^0$.

$2\pi R \rightarrow 360^0,$

$\Rightarrow \frac{2\pi R}{360} \rightarrow 1^0 \text{ and } \frac{2\pi RD}{360} \rightarrow D^0$

i.e. $\frac{2\pi RD}{360} = 30.5$; Since arc length of 30.5m corresponds to $D^0$.

$D = \frac{1746.813}{R} = \frac{1750}{R}$

.................

.....(1.1)
hand (RH) curve is there if the change in direction of the curve is in clockwise direction when seen in the direction of travel in multiple lines or in the direction of increasing kilometers in case of single line.

1.3 Versine of a Curve: The curves are identified by the degree or radius. But both of these are difficult to measure in the field due to the very large radii of the railway curves. The versine is a very easy measurement which can be used in the field for measurement and rectification of the geometry. **The versine of a curve stands for the ordinate from the midpoint of a chord on the curve**. Figure 1.2 illustrates the concept of versine in a curve. In arc AB, if the chord AB of length, C, on a circle having radius R is considered, then the ordinate EF will be the versine, v. Let us extend arc AB to complete the circle of radius R (shown in figure 1.2) and extend the versine FE to D through centre of circle O.

Now, circle has a property that if two chords intersect, the product of the two parts is equal for both the chords. Using

---

The versine mentioned here shall not be confused with versed sine which is also sometimes called versine (see para 4.3.2, Chapter IV)
this property for the chords DF and AB, \( \text{DE} \times \text{EF} = \text{AE} \times \text{EB} \)
i.e. \( (2R - v) v = (C/2) (C/2) \)
i.e. \( 2Rv - v^2 = C^2/4 \). Since value of versines is normally in millimetres or centimeters, whereas value of \( R \) is in metres, \( v^2 \) is very small compared with \( 2Rv \), and can be neglected.
i.e. \( 2Rv = C^2/4 \)
i.e. \( v = \frac{C^2}{8R} \)  
\[ \text{(1.2)} \]

On Indian Railways, measurement of curve is done at nominated points on the curve, which are usually paint marked on the track. These points are called stations. The procedure for measurement of versines is explained in Chapter II, para 2.2.

1.4 Movement of Vehicle on Curved Track: When a vehicle moves over the curved track, following are to be achieved:

1.4.1 Continuous change in direction: The change in alignment of a vehicle on a curved track is done by the rails. The rail closer to the center of curve is called inner rail and the rail farther away from the center of curve is called outer rail. The leading wheel of a bogie (or trolley) in case of a bogied vehicle and the leading wheel of a vehicle in case of a four-wheeler vehicle moves with positive angularity, attacking the outer rail of the curve. The change in direction of outer rail causes the wheel to change direction (see figure 1.3). Due to this, there are large lateral forces on the track as well as vehicle.

![Fig. 1.3](image-url)
1.4.2 Movement without slip: On a curved track the length of outer rail is more than the length of inner rail, therefore the outer wheel must travel with a larger radius as compared to the inner wheel (Figure 1.4). Therefore, a mechanism is required to ensure that the outer wheel does not slip during movement on a curve. The actual vehicle movement on curve is quite complicated but we can consider that the vehicle traverses the curved path without appreciable slip due to coning in the wheels, which causes larger diameter to travel on the outer rail and smaller diameter on the inner rail by slight shifting of the center of gravity of the vehicle towards the outer rail. This is shown in figure 1.5 below.

\[
\begin{align*}
R - \frac{G}{2} & \quad \text{Outer Rail} \\
R + \frac{G}{2} & \quad \text{Inner Rail}
\end{align*}
\]

Where \( r \) is radius of wheel at center

Fig. 1.4

Where \( 2r + \Delta r \) and \( 2r - \Delta r \)

Fig. 1.5 (Section A-A in fig 1.3)
However, the above mechanism is possible only up to a certain radius of the curves and if the radius is sharper, the outer wheel will skid and inner wheel will slip in order to ensure that the axle moves as a unit in the desired direction. To reduce the wear and tear due to this, slack gauge is permitted on sharp curves (see para 2.6 for details.)

1.4.3 Forces on a Vehicle During Movement on Curve: The vehicle passing over the curve continuously changes its direction over a curve. Due to inertia, the vehicle tends to continue moving in the straight line but the forced change in direction of the movement by track gives rise to lateral acceleration* acting outwards which is felt by the vehicle and all passengers/things inside. This acceleration is called Centripetal Acceleration and the force due to the same is called Centrifugal Force.

Therefore, in the railway track, the vehicle/consignment in the vehicle will experience lateral forces when the vehicle travels in a curved path and if the curve is not designed or maintained to proper geometry, the unbalanced forces in the lateral direction will further go up. These forces will lead to higher maintenance efforts and the comfort of the passengers as well as safety of the vehicles on the curve may also be affected.

1.4.4 Managing the centrifugal force: To counteract the effect of centrifugal force, the raising of outer rail with respect to inner rail\(^2\) is done. This raising of outer rail of a curve with respect to inner rail is referred to as cant or superelevation, C. Due to the vehicle being on slope, a component of weight starts acting towards the center of curve. This component, called Centripetal Force, acts opposite to the direction of the centrifugal force**. The forces on a vehicle on the curve having cant is shown in Figure1.6.

If, in some situation, outer rail of curve is lower than inner rail, the cant is said to be negative cant. In this case, the

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*Strictly speaking, this is a pseudo-force, which is required to apply Newton's laws of motion in a system which is accelerating due to changing velocity resulting from a constant change in direction while train travels on a curve. For simplicity sake, it is referred to as force
** Centripetal force is an actual force.
2 Para 404 of IRPWM
centripetal force will act in the same direction as centrifugal force.

1.4.5 Quantitative Analysis of Forces on Vehicle on a Curve: If a vehicle having mass M (weight, \( W = M \times g \)) is moving on a curve of radius \( R \) with a speed \( V \), the centrifugal force experienced by the vehicle comes to \( MV^2/R \). This force is acting at center of gravity of the vehicle parallel to the plane of rotation and away from the center of the curve which will be changing with changing \( V \). Depending on cant, a constant centripetal force will be acting towards the center of curve. When the two forces acting in the lateral direction match with each other, the vehicle is in equilibrium as far as lateral forces are concerned. In this situation, any person sitting inside the vehicle will not be able to differentiate between the motion on a straight or a curve due to the absence of lateral forces. The cant at which the lateral forces are in equilibrium on a curve is called equilibrium cant \( (C_{eq}) \). While calculating centrifugal force, Radius should be measured in inclined plane. But practically Radius is measured in horizontal plane. Since angle \( \theta \) is less and radius is large, the Radius 'R' in inclined plane to approximated to be equal to Radius 'R' in horizontal plane. Hence,
\[
\frac{MV^2}{R} = w\sin\theta
\]

\[
\Rightarrow \frac{MV^2}{R} = (M \times g) \frac{C_{eq}}{G}
\]

Where \( G \) = Dynamic gauge or centre of contact of wheel tread over the rails, 1750 mm for BG and 1058 mm for MG

\( C_{eq} \) = Equilibrium cant corresponding to speed \( V \).

\[
:\therefore C_{eq} = \frac{GV^2}{gR} \quad \text{.................................}(1.3)
\]

1.4.6 Equilibrium speed \((V_{eq})\): It is the speed on a curve at which the actual cant exactly counteracts the centrifugal force. If all the trains passing over a curve move at same speed, the speed shall be chosen as the equilibrium speed and the curves designed accordingly.

1.4.7 Computation of equilibrium speed: If we provide the equilibrium cant corresponding to the speed of movement of trains, the trains will be moving on the curve without any net lateral force and following benefits are there:

- Load is equal on both rails.
- The wear on both rails is equal
- Maintenance of track geometry is easier.
- Fittings and fixtures are subjected to less stresses and wear and tear.
- The passengers are not discomforted.

Therefore, the desirable situation is that the actual cant shall be provided for the speed of train on the curve. However, in a situation most prevalent on Indian Railways, the various trains move at different speeds. The same track is shared by the goods trains, local trains, mail/express trains and the Rajdhani/ Shatabdi trains & other high speed train etc. This means that speed for which the actual cant is to be provided has to be decided based on the numbers of trains and their speeds of travel on the curve.

These trains will experience cant more than or less than that required for that particular speed. Trains which are not moving at the equilibrium speed experience either of the two conditions:
a) **Speed of Vehicle More than Equilibrium Speed**: For a vehicle moving at a higher speed than equilibrium speed, the cant required is more than actual cant provided and the difference between the two is called **cant deficiency, C_d**. Such a vehicle experiences outwards lateral force as the centrifugal force is more than centripetal force and in this situation, the wheel load on the outer rail is more than the wheel load on inner rail.

b) **Speed of Vehicle Less than Equilibrium Speed**: Cant required for a vehicle moving at a lesser speed than equilibrium speed is less than actual cant provided. This difference between the cant actually provided and the cant required for the actual speed is called **cant excess, C_{ex}**. Such a vehicle experiences inwards lateral force as the centrifugal force is less than centripetal force and in this situation, the wheel load on the inner rail is more than the wheel load on outer rail.

Therefore, the equilibrium speed shall be so chosen that the effect of trains at speed more than equilibrium speed is balanced by the effect of trains at lesser speeds.

As per IRPWM\(^3\), the equilibrium speed for which cant is provided in a curve is to be decided by CE, after the following factors have been considered:

1. The sectional speed limit;
2. Proximity of permanent speed restrictions such as junctions, stopping places etc;
3. Track Gradient which may cause a reduction in the speed of freight or slow moving passenger trains without appreciable reduction effect on the speed of fast trains;
4. The relative importance of various types of traffic.

For this purpose, the entire section may be divided into a certain number of such sections with a nominated equilibrium speed for each sub-section. On lines which carry predominantly slow moving, heavy freight traffic, it may be necessary in exceptional circumstances to limit the cant in order to avoid excessive wear on the lower rail and this may result in speed restriction on faster trains. Each line of a

\(^3\) Para 406(b) of IRPWM
double line section should be considered individually.

To determine the equilibrium speed, local conditions have to be seen as above. The IRPWM does not specify any objective criteria but a formula, called **Russian formula**, (equation (1.4)) is a very good method for calculating the equilibrium speed. Example 3.9, chapter III illustrates the use of this formula.

\[
V_{\text{eq}} = \sqrt{\frac{\sum_{i=1}^{m} n_i W_i i^2}{\sum_{i=1}^{m} n_i W_i}} \quad \text{..............................................(1.4)}
\]

where:

- \(V_{eq}\) = Proposed equilibrium speed
- \(n_i\) = Number of trains in \(i^{th}\) speed group
- \(W_i\) = Weight of each train in \(i^{th}\) speed group
- \(V_i\) = Speed of each train in \(i^{th}\) speed group
- \(m\) = Number of groups

**1.5 Transition Between Straight and Circular Curve:** Let us examine the situation where a vehicle enters a curve. If the circular curve directly follows the straight, the radius of the track will suddenly reduce from infinity in straight to \(R\) in the circular curve, thereby producing a jerk due to sudden introduction of centrifugal and centripetal forces. This will not be desirable from the passenger comfort as well as safety considerations.

Also, the track in the straight is level and whereas the outer rail in the circular curve is raised with respect to inner rail by an amount equal to cant, \(C_a\). If we raise the track at a single point, this can lead to unsafe condition for the vehicle. To achieve the comfort and safety on the entry of the curve, the change in radius and introduction of cant is to be done gradually in the track. This is accomplished through a small easement curve of varying curvature inserted between straight and circular curve, called **transition or spiral**. The transition curve also helps in managing
the centrifugal and centripetal forces on the vehicle, as these are dependent on the radius of the curve at any point.

It is to be ensured that corresponding to the curvature (which is inverse of radius i.e. Curvature=1/Radius), the superelevation or cant is provided in a transition so that the centripetal force is available proportionate to the centrifugal forces. Design of transition is given in para 1.8.

1.6 Deciding curve parameters:

1.6.1 Deciding Radius of curve (R): As discussed above, the curve is introduced between two sets of straight alignment. The straight alignment or tangent track is known from the survey keeping in view the various factors listed at Para 1.1 above. A large number of curves of varying radii are possible to be introduced between the straights (Figure 1.7) and we have to choose the most suitable curve for our purpose.

The flatter curves (having more radius) are longer. Sometimes, these may be uneconomical to be laid or not at all feasible due to the site features. The sharper curves (having less radius), the length of curve is less but the lateral forces on
the vehicles are more, resulting in accelerated wear of the rails/wheels and more strain on the fastenings. Maintaining sharper curves is more difficult and expensive as compared with the flatter ones. The minimum radius to which curves can be designed on any particular section is determined by the speed potential desired, maximum values of cant, cant deficiency and cant excess permissible, and the negotiability of the curve as per the design of vehicles. If we provide a curve sharper than the minimum required, a permanent speed restriction may be required in the section. Therefore, for a given situation, curve chosen should be optimised between flat radius and loss or length. The issue has been discussed in more detail in paras 4.1 and 4.3, chapter IV.

The absolute minimum radii on Indian Railways are laid down in SOD as under:

BG – 175 m
MG – 109 m
NG – 44 m

1.6.2 Deciding Cant or Superelevation:

1.6.2.1 Limiting values of Cant \((C_{a_{\text{max}}})\): When the track is laid to a cant, the sleeper is in slope, resting on the ballast cushion. The vehicle on the canted track also assumes a tilted position and in standing condition, the vehicle will have lesser force on the outer rail and more force in the inner rail. The following aspects of track-vehicle interaction are required to be considered:

a) Overturning about inner rail: In case of a very highly canted curved track, it is possible that a vehicle traveling over the curve at a very low speed or standing over it may overturn about the inner rail towards the inside of the curve. Such overturning can occur on account of the combination of the following adverse factors:

- Absence of centrifugal force (due to very slow speed or NIL speed) and presence of large centripetal force due to high cant.

---

4 Item 2, Chapter I, Schedule I of SOD
- Wind pressure blowing on the vehicle towards the inside of the curve.
- Vibration and other disturbing forces.

We shall consider the case when the vehicle is stationary i.e. when \( V=0 \). Taking moments about the inner rail and adopting a factor of safety of 3, the maximum cant that can be permitted is \( \frac{G^2}{6H} \) where \( H \) is the height of C.G. of the vehicle above rail level. For typical cases of rolling stock, this works out to 304 mm for BG and 146 mm for M.G.

**b) Derailment by Flange Climbing:** The above scenario of the vehicle overturning about the inner rail does not consider the lateral force which guides the wheel to take the curved path for a vehicle about to start while standing on a curve (see figure 1.3) Considering this force, it is more likely that before the condition for overturning about the inner rail is reached, derailment by wheel climbing would occur. When a vehicle stands on a highly canted track, the reaction on the inner rail is more than that at the outer rail. The load on outer rail reduces compared to the nominal wheel load. This is called off loading of the outer wheel. Just at the moment a vehicle starts moving, the lateral forces \( Y \) which guide the wheels along the curve are maximum (as force is required to overcome the inertia of vehicle standing as well as the rolling friction on the vehicle). These forces cause the \( Y/Q \) ratio (lateral flange force to instantaneous wheel load ratio) for the outer leading wheel to assume adverse proportions, making it prone to derailment. Empty vehicles have lower dead weights and these are the most prone to derailments on a curve.\(^5\)

**c) Maintainability:** The track in highly canted condition is more difficult to maintain as the sleeper is lying in a slope, and the end reaction from the sleeper starts coming into picture. This will make the retention of the ballast under the inner rail difficult. There is, thus, a physical limit to the actual cant beyond which the track parameters can not be easily maintained.

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\(^5\) For detailed study, please refer to the book “The Investigation of Derailments”, March 2007, published by IRICEN.
Keeping in view the above aspects, a thumb rule normally adopted for the limiting value of cant is G/10 all over the world railways. However, on sections carrying predominantly passenger traffic, most of the trains are moving with the cant deficiency, thus pushing the vehicle towards the outer rail. Therefore, on high speed passenger lines on Japanese and French Railways, the limit for cant is adopted as G/8, limiting the radius of curvature to 4000 m. The maximum cant allowable on Indian railways is listed in table 1.1 below.

Table 1.1: Maximum value of cant allowed on Indian Railways

<table>
<thead>
<tr>
<th>Gauge</th>
<th>Maximum Cant Allowed normally</th>
<th>Maximum Cant Allowed with special permission of CE</th>
</tr>
</thead>
<tbody>
<tr>
<td>BG Group A, B, C</td>
<td>165mm</td>
<td>-</td>
</tr>
<tr>
<td>BG Group D,E</td>
<td>140mm</td>
<td>-</td>
</tr>
<tr>
<td>MG</td>
<td>90mm</td>
<td>100mm</td>
</tr>
<tr>
<td>NG</td>
<td>65mm</td>
<td>75mm</td>
</tr>
</tbody>
</table>

Note: (1) Maximum Cant of 185 mm may be assumed for all permanent structures etc by the side of curves on new constructions and doublings on Group ‘A’ routes having potential for increasing the speed in future. The transition length should also be provided on the basis on 185mm cant for the purpose of planning and layout of curve.

(2) Cant for each curve shall be specified and indicated on web of inside of curve, to the nearest 5 mm. The superelevation to be provided shall be specified when the line is originally laid and thereafter altered only with the prior approval of Chief Engineer.

1.6.3 Deciding Cant Deficiency ($C_d$) : If the vehicles move at equilibrium speed, there is no lateral force. However it has been found out experimentally that the passengers sitting inside the train have some capacity to bear the lateral forces without getting discomforted. Therefore, to take the advantage

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6 IRPWM para 406 (d)  
7 IRPWM para 406(d)(1)(i)  
8 IRPWM para 406(e)
of this, we permit trains to travel at speeds higher than the equilibrium speed to which cant has been actually provided. In such a situation, the centrifugal force is more than the centripetal force (and thus acceleration) acting outwards may causes discomfort to the passengers. Cant deficiency is an index of this discomfort felt by the passengers. If the cant deficiency is kept within limits, the passengers are not unduly discomforted and their time is saved due to higher speed of travel.

To decide the limits on the values of cant deficiency following considerations are there:

a) Safety against overturning;
b) Safety against derailment
c) Maintainability; and
d) Comfort of passengers.

Considerations (a) to (c) permit much larger values of cant deficiency than those dictated by consideration of comfort to the passengers. Based on calculations and actual trials, it is generally accepted that unbalanced lateral acceleration should be within 0.4 to 0.7m/sec$^2$. From this guideline, it is possible to work out, for any given track gauge, the corresponding values of cant deficiency, based on the relationship given below:
From Eq 1.3, Equilibrium Cant, $C_{eq} = \frac{G V^2_{eq}}{g R}$

and Actual Cant, $C_a = \frac{G V^2}{g R}$

Where $V_{eq}$ is Equilibrium speed and $V$ is actual speed of the train. Subtracting the above two equations,

$$C_a - C_{eq} = \frac{G V^2}{g R} - \frac{G V^2_{eq}}{g R},$$

Since $C_d = C_a - C_{eq}$,

$$C_d = \frac{G}{g} \left( \frac{V^2 - V^2_{eq}}{R} \right)$$

$$C_d = \frac{G}{g} (\Delta_p),$$

Where $\Delta_p$ is the unbalanced lateral acceleration in m/sec$^2$

ie $\Delta_p = \frac{C_d}{g} g$ ............................................................... (1.5)

Substitute $G=1750mm$ in eq (1.5) for BG, the limits of $C_d$ will vary from 71 mm to 125 mm. Corresponding figures of $C_d$ for MG ($G=1058mm$) will vary from 43 mm to 75mm.

The above calculations are valid for a rigid vehicle. In actual practice, the vehicles have springs, which compress variably under different loads. Due to the unbalanced lateral acceleration, the load on outer rail is higher than on inner rail. The springs on outer wheel, therefore, compress more than the springs on inner wheel (see figure 1.9).

This causes vehicle body to tilt outwards more than the tilt calculated considering the cant and therefore, the actual unbalanced lateral acceleration experienced by passengers is more than theoretically calculated. Therefore, the limits of cant deficiency to be chosen shall be less than the maximum theoretical values calculated above. On this consideration, the limit-
ing values of $C_d$ have been chosen on IR, are given in table 1.2.

### Table 1.2: Limits of $C_d$ adopted on the Indian Railways

<table>
<thead>
<tr>
<th>S No.</th>
<th>Route</th>
<th>Cant Deficiency Allowed</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>For speeds in excess of 100 KMPH on group A and B routes for nominated rolling stocks and routes with permission of Chief Engineer</td>
<td>100 mm</td>
</tr>
<tr>
<td>(b)</td>
<td>For BG routes not covered by above</td>
<td>75 mm</td>
</tr>
<tr>
<td>(c)</td>
<td>MG</td>
<td>50 mm</td>
</tr>
<tr>
<td>(d)</td>
<td>NG (762 mm)</td>
<td>40 mm</td>
</tr>
</tbody>
</table>

*Note: For higher speeds, it is not desirable to have high cant deficiency as the discomfort felt by the passengers at the higher speed becomes too much and it is desirable to limit the cant deficiency to 75 mm for speeds higher than, say, 160 KMPH.*

### 1.6.4 Deciding Cant Excess ($C_{ex}$): When a train travels around a curve at a speed lower than the equilibrium speed, the centripetal force is in excess of the centrifugal force. The available cant is more than the cant required to counteract this

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9 Para 406(2) of IRPWM
centrifugal force and the vehicle experiences a condition of **cant excess**. In this situation, the inward Centripetal Force is more than the outward Centrifugal Force and the net lateral force will be inwards. The reaction on the inner rail will be more than the reaction on the outer rail. (See figure 1.10)

If any train stops due to any reason on the curve, the speed becomes minimum, i.e. zero. For such a train, actual cant provided becomes the cant excess. In the para 1.6.2.1 above, we have analyzed the situation when there is minimum speed on a curve (zero). The effects on safety and comfort have both been analyzed therein, and the maximum value of the actual cant has been decided on these considerations.

On the mixed traffic conditions prevailing on IR, the Cant excess criteria takes into account the effect of the slow moving goods trains on the maintenance of the curve. On curves, the slow moving goods trains put more load on the inner rail as compared to the outer rail. These cause more wear on the top table of the inner rail. The wear caused by the passenger/ fast moving vehicles is in the gauge face of the outer rail. Since the wear surface on the inner and outer rails are different, it will be desirable to be able to interchange the rails after the limit of wear is reached. However, since the load and intensity of goods train is quite high as compared to

**Fig. 1.10**

![Diagram showing centrifugal and centripetal forces](image_url)
the passenger trains, if the cant excess for the goods trains is higher, the wear in the inner rail will be very high and the wear on the outer rail will be comparatively lesser. This will necessitate the renewal of the inner rail alone. This will be uneconomical for track maintenance. **In order to achieve economy, the cant excess on curves is limited for the booked speed of goods trains in the section.**

From above discussion, it can be concluded that for sections carrying less goods traffic, higher value of cant excess should be allowed. And for sections carrying predominantly freight traffic, value of cant excess allowed should be kept low. Some of the world railways have directly related the permissible limits of cant excess to the intensity of freight traffic on a particular section. On IR, though a single value permissible value has been specified for all the routes, it has been specified that for routes carrying predominantly goods traffic, the cant excess shall be preferably kept low. **The maximum permissible limits have been set at 75 mm and 65 mm respectively for BG & MG**. For calculating the cant excess, the booked speed of goods trains in the section has to be considered.

**1.7 Calculating Speed on a Curve:** If \( C_a \) is the cant actually provided in the track, for a vehicle traveling on the curve at a speed higher than equilibrium speed, the cant deficiency \( C_d \) is given by

\[
C_d = \frac{GV^2}{gR} - C_a \quad \ldots(1.6)
\]

\[
\therefore \frac{GV^2}{gR} = C_a + C_d
\]

\[
\Rightarrow V = \sqrt{\frac{g}{G} \times \sqrt{(C_a + C_d) \times R}}
\]

Substituting \( g = 9.81 \text{ m/sec}^2 \), \( G = 1750 \text{ mm} \) and adopting \( C_a \) in mm, \( V \) in KMPH, and \( R \) in m, and accounting for the units, the equation reduces to

\[
V = 0.27 \times \sqrt{(C_a + C_d) \times R} \quad \ldots(1.7)
\]

\[1^0 \text{ Para 406 (3) of IRPWM} \]

\[1^1 \text{ Para 405(a) of IRPWM} \]
As discussed in para 1.6.2.3 above, the goods trains moving in the section at the booked speed, $V_g$, experience a condition of cant excess. The cant excess is given by

Using equation (1.3),

$$C_{ex} = C_a - \frac{G V_g^2}{gR} \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots 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Desirable features of a transition are:}

a) In order that there is no jerk at the either end of the transition, the transition curve must be tangential to the straight as well as the circular curve.

b) It shall ensure a gradual increase of curvature (= 1/Radius) from zero at tangent point (Radius = \infty) to the specified curvature 1/R for the circular curve.
c) Correspondingly, in the transition portion, the cant shall be increased from zero to $C_a$.

To meet the above objectives, the transition curve is introduced between straight and circular curve such that 50% of the same is in straight and 50% in circular portion.

Ideal shape of a transition is one which changes its radius from infinity to that of circular curve at a constant rate of change with distance traveled. (Figure 1.11)

The equation of this curve is:

$$l \propto \frac{1}{r} \text{ i.e. } l \cdot r = \text{constant} \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots (1.9)$$

where $l$ is the distance traveled on the transition curve and $r$ is the radius at the distance $l$ from the start of the transition curve. The curve described by the eq. (1.9) is called a clothoid or Euler spiral.

Using the spiral as the shape for railway curves requires use of infinite series for determining location of each point. This is cumbersome and requires lots of calculations. Such a curve used to be difficult to lay in field in the pre-computer era, hence the shape adopted for transition curves on Indian Railways is cubic parabola.

In a cubic parabola, the rate of change of curvature is uniform with the distance traveled along x-dimension and not the distance traveled along the curve. There is not much difference between a cubic spiral and a cubic parabola until
the throw is up to 4m or the deflection angle is up to $12^0$. The curves adopted on IR have large radii and therefore, the cubic parabola is good enough approximation to be used as transition curve in place of a spiral for majority of the curves. Figure 1.12 shows the paths taken by a cubic parabola and a spiral.

1.8.3 **Cubic Parabola as transition:** Let us draw the transition curve again, keeping the tangent track horizontal, as shown in figure 1.13.

For the cubic parabola, the equation (1.9) gets converted to

$$x \times r = \text{Constant.}$$

Now, at $x = L$, $r = R$ therefore, $RL = \text{Constant.}$

i.e. $x \times r = RL$ and $\therefore \dfrac{1}{r} = \dfrac{x}{RL}$

If $\phi$ is the angle subtended by the tangent at any point on curve
(see figure 1.13), the curvature is given by \( \frac{1}{r} = \frac{d\phi}{dx} \)

Integrating, we get \( \phi = \frac{x^2}{2RL} + \text{Constant 1} \)

At \( x = 0 \), transition is tangential to the tangent track, so \( \phi = 0 \)

\( \therefore \text{Constant 1} = 0 \) in above equation

i.e. \( \phi = \frac{x^2}{2RL} \) ...........................................................................(1.10)

Now, \( \phi = \frac{dy}{dx} \) \( \therefore \phi = \frac{dy}{dx} = \frac{x^2}{2RL} \).

On integrating, \( y = \frac{x^3}{6RL} + \text{Constant 2} \)

At \( x = 0 \), \( y \) is 0; so the \( \text{Constant 2} \) in above equation becomes 0

\( \therefore \) The equation reduces to \( y = \frac{x^3}{6RL} \) ...........................(1.11)

The equation (1.11) is the equation of a cubic parabola, which is used as transition curve on IR.

1.8.4 Deciding Rate of change of cant deficiency, \( r_{cd} \):

When the trains move at speed higher than the equilibrium speed the cant deficiency exists, which results in net outwards lateral acceleration acting on the vehicle. The total amount of the lateral acceleration is discussed in para 1.6.2.2 however the same must be introduced gradually else there can be discomfort to the passengers. The rate of Change of unbalanced lateral acceleration (ULA) has a direct bearing on passenger comfort. Therefore to avoid the passengers from being discomforted, the rate of change of ULA (and hence cant deficiency) over the curve has to be kept within limits. The limit for this has been worked out as 0.03g m/sec\(^3\) i.e. 0.3 m/sec\(^3\), approximately.
As discussed in para 1.6.3 above, Cant Deficiency at end of transition:

\[ C_d = \frac{G}{g} (\Delta p) \]

At start of transition, the vehicle is on the straight,

\[ \therefore \text{Cant Deficiency} = 0. \]

For a vehicle traveling at speed \( V \), time of travel on the transition curve of length \( L \) is \( L/V \).

\[ \therefore \text{The rate of change of cant deficiency over the transition:} \]

\[ \frac{C_d - 0}{L/V} = \frac{G}{g} \left[ \Delta p \left( \frac{L}{V} \right) \right] \]

i.e. \( r_{cd} = \frac{G}{g} \) [rate of change of unbalanced lateral acceleration]

i.e. \( r_{cd} = \frac{1750\text{mm}}{9.81\text{m/sec}^2} \left[ 0.3\text{ m/sec}^3 \right] \approx 53.5\text{ mm/sec} \), rounded off as 55 mm/sec.

The 55 mm/sec is the maximum rate of change of lateral acceleration which can be permitted. From the consideration of passenger comfort, a lower value is preferred and the desirable rate of change of cant deficiency is laid down as 35mm/sec.

Similarly for MG, \( r_{cd} = 0.3\text{mm} \times \frac{1058\text{ mm}}{9.81\text{ m/sec}^2} \]

\[ = 32.35\text{mm/sec} \approx 35\text{mm/sec}. \]

Table 1.3 : Limits for \( r_{cd} \) laid down on Indian Railways\(^{12}\)

<table>
<thead>
<tr>
<th>Gauge</th>
<th>Normally</th>
<th>In exceptional circumstances</th>
</tr>
</thead>
<tbody>
<tr>
<td>BG</td>
<td>35mm/sec</td>
<td>55mm/sec</td>
</tr>
<tr>
<td>MG</td>
<td>35mm/sec</td>
<td>35mm/sec</td>
</tr>
</tbody>
</table>

Note: 1. The values of \( r_{cd} \) from safety considerations are

\(^{12}\) Para 407 (3) of IRPWM.
quite on higher side, and the values mentioned above are all from the passenger comfort point of view.

2. The maximum rate of change of cant and cant deficiency is limited to 55 mm/sec for speeds upto 160 KMPH.\textsuperscript{13}

1.8.5 Deciding Rate of change of actual cant $r_{ca}$: Similar to the change of cant deficiency, the rate of change of actual cant also has a bearing on comfort. This is applicable when the vehicle is traveling over the curve at speeds lesser than the equilibrium speed. For the same levels of comfort, a higher rate of change of actual cant can be permitted than the rate of change of cant deficiency. However, at present, the limits for $r_{ca}$ are kept the same as for $r_{cd}$ on Indian Railways.\textsuperscript{14} The values given in the Table 1.3 are used for $r_{ca}$ also on Indian Railways.

1.8.6 Deciding Cant Gradient: The gradual change of curvature is accompanied by gradual change in superelevation in a transition curve. The changing superelevation induces ‘twist’ in the track. The twist is defined as the algebraic difference of the cross levels (i.e. sign of the cross-levels is to be taken into consideration) at two points in track divided by the distance between the two points (See Figure 1.14).

![Figure 1.14: Plan of a vehicle on track](image)

If the cross levels of the track at the locations of the wheels shown in figure 1.14 are:

At A: 4 R, (The right rail is 4 mm lower than left rail, when seen in the direction of travel)
At B: 4 L (The left rail is 4 mm lower than the right rail)
At C: 10 L (The left rail is 10 mm lower than the right rail)
At D: 20 L (The left rail is 20 mm lower than the right rail)

Now, the twist has been defined as the rate of change of cross levels over the wheel base.

Since the wheel base is 2 m in this case as shown in figure 1.14, the twist can be found out as:

In the leading wheel set, between wheelset D and wheelset C: $(20L-10L) \text{ mm}/2 \text{ m} = 10/2 \text{ mm/m} = 5 \text{ mm/m}$

In the trailing wheel set, between wheelset B and wheelset A: $(4R-4L) \text{ mm}/2 \text{ m} = (4R-(-4R))/2 \text{ mm/m} = 8/2 \text{ mm/m} = 4 \text{ mm/m}$

\textsuperscript{13} RDSO letter no 68/WDO/SC/1 dated 15.4.1968
\textsuperscript{14} Para 407 (3) of IRPWM.
The twist physically means that three points of the contact between vehicle and track are in one plane and fourth is out of the plane. Just as a table with a leg smaller than others rocks and one of the legs loses contact with ground, the twist causes one of the wheels to off load and even lose contact with the rail. The offloading makes a vehicle prone to derailment\(^{15}\), so twist is one of the most important factors affecting safety of the vehicles. On a transition curve, therefore the twist introduced has to be kept well within limits.

Cant gradient, \(i\), refers to the amount by which cant is increased or reduced in a given length. It is either expressed in unit less terms such as 1 in 1000 (indicating 1mm gain/loss in superelevation in every 1000mm length of transition) or with units such as 1.4 mm/m indicating 1.4 mm gain/loss in superelevation in every 1m length of transition.

**The cant gradient shall be limited to 1 in 720 or 1.4 mm/m on Indian Railways for BG and MG. However in exceptional circumstances, the cant gradient may be relaxed to 1 in 360 or 2.8mm/m in BG\(^{16}\).**

**Note for Maintenance:** *Since the total value of the allowable twist from safety considerations is fixed, the permissible or design value of cant gradient reduces the extent to which the further track irregularities can be permitted during maintenance. Therefore, to give margin to the maintenance engineer for meeting the laid down track parameters in service, the twist introduced through cant gradient shall be kept as low as possible, and it is preferable to lay the curves to recommended values of cant gradient rather than the exceptional values laid down.*

Cant gradient and rate of change of cant are interlinked. The chosen cant gradient should be such as to ensure that the rate of change of cant does not exceed 35 mm per second for BG and MG, or under exceptional circumstances, does not exceed 55 mm per second on BG. On BG, for speeds upto 130 KMPH, a cant gradient of 1 in 720 would mean \(r_{ca} = 50.2\)

\(^{15}\) See paras 3.7 and 7.2.3 in the IRICEN book on “Investigation of Derailment” for detailed discussion on the subject.

\(^{16}\) Paras 407(2) and 407(3) of IRPWM.
mm/sec, which is more than normally permitted 35 mm/sec, though less than the exceptionally permitted 55 mm/sec. In future layouts on BG where high speed is contemplated, a cant gradient of 1 in 1200 is recommended\textsuperscript{17}, so that even at a speed of 160 KMPH, the value of \( r_{ca} \) will be limited to 37 mm/sec.

1.8.7 Finding Length of transition: The length of transition required is to be worked out based on the criteria given in para 1.8.4, 1.8.5 and 1.8.6 above. The length of transition to be provided shall be worked out as below:

1.8.7.1 Desirable Length based on criterion of rate of change of cant deficiency:\textsuperscript{18}
If we consider the length of transition and the speed of travel, the time of travel of a vehicle on the transition curve is \( \frac{L}{V} \).

And from the rate of change of cant deficiency criteria, time of travel on the curve is \( \frac{C_{d}}{r_{cd}} \).

Equating, \( \frac{C_{d}}{r_{cd}} = \frac{L}{V} \), \( \therefore L = C_{d} * \frac{V}{r_{cd}} \)

Taking value of \( r_{cd} \) as 35 mm/sec, and taking care of the units,

\[
L = 0.008 \times C_{d} \times V_m \tag{1.12}
\]

where \( C_{d} \) is in mm and \( V_m \) is maximum speed on curve section in kmph and \( L \) is in metres.

1.8.7.2 Desirable Length based on Criterion of rate of change of actual cant:\textsuperscript{18}
Since the value of \( r_{ca} \) and \( r_{cd} \) are similar,

\[
L = 0.008 \times C_{a} \times V_m \tag{1.13}
\]

where \( C_{a} \) is in mm and \( V_m \) is maximum speed on curve section in kmph and \( L \) is in metres

1.8.7.3 Length based on Criterion of cant gradient\textsuperscript{18}
If \( i \) is the cant gradient permissible, the length of the transition shall be

\[
L = i \times C_{a} \tag{1.14}
\]

\textsuperscript{17} RDSO letter no 68/WDO/SC/1 dated 15.4.1968.

\textsuperscript{18} Para 407(1) of IRPWM.
Desirable value of \( i \) is 1 in 720. Substituting the same and taking care of the units, \( L = 0.72^* C_a \) \…………………………(1.14)
where \( C_a \) is in mm and \( L \) is in metres.

The desirable length of transition to be provided shall be more than that worked by equations (1.12), (1.13) and (1.14) above.

1.8.8 Length of transition under exceptional circumstances on BG: Geometry dictates that the provision of transition at the ends of a circular curve is accompanied by shifting of the circular curve inwards (The amount of shift can be calculated from the equation (1.21)). The concept of shift has been explained further in para 1.11 in this chapter. Based on site conditions such as bridges/ tunnels/ other constructions etc sometimes, the inward shifting of the curve is restricted. In such a case, the desired length of the transition may not be possible to be provided. Such circumstances may be treated as exceptional circumstances and the reduced length of transitions worked out as below may be provided:

1.8.8.1 Minimum Length based on rate of change of cant actual/ cant deficiency considerations\(^{19}\): The rate of change of cant actual/ cant deficiency permitted normally is 35 mm/sec, and 55 mm/sec under exceptional circumstances. Therefore, the length of transition worked out as per equation (1.12) and (1.13) shall be reduced to 2/3\(^{rd}\) under exceptional circumstances.

1.8.8.2 Minimum Length based on cant gradient considerations: The cant gradient permitted under exceptional circumstances is 1 in 360. Therefore, the length of transition worked out as per equation (1.14) shall be reduced by 1/2 under exceptional circumstances.

Under exceptional conditions, the minimum length of transition to be provided shall be more than that worked as per 1.8.8.1 and 1.8.8.2 above.

1.8.9 Curves with Length of Transition Less Than Minimum Required: If the space available at site is such that even the length of transition worked out for the exceptional cir-

\(^{19}\) Para 407(3) of IRPWM.
cumstances cannot be provided, reduced transition length shall be provided to the extent possible. This would, however, restrict the amount of cant/ cant deficiency which can be provided due to the limitation of maximum $r_{ca}$ and $r_{cd}$. This will necessitate speed restriction to be imposed. The speed potential of the curve shall be found out by reverse calculations for the available length of transition.

1.9 Ideal Versine and Cant Diagram of a Curve: The curvature of the transition curve by double differentiating eq 1.11 can be expressed as:

$$\frac{d^2y}{dx^2} \propto \frac{1}{r} \frac{x}{2RL}.$$  

Therefore, the curvature (and hence the versines) increase linearly with the distance on the transition curve. It is therefore, desirable that the cant shall also be introduced linearly with the distance on the transition. The curvature is constant in the circular portion and so is the cant provided have a fixed value for the circular portion of the curve. The ideal versine/ cant diagram for circular curve with transitions having cubic parabolic shape shall have a trapezoidal shape as shown in figure 1.15.

![Fig. 1.15: Trapezoidal distribution of versines and cant in a typical curve](image)

Ca: Cant Actual in circular portion. V: Versine in the circular portion.

1.10 Types of curves: Different types of curves in the track are:

- Simple curves
- Compound curves
- Reverse Curves
1.10.1 Simple Curve: A simple curve is a single circular curve with uniform radius throughout the length, joining two tangent tracks (see figure 1.16). Transition curve may be there at either end junction of the curve with tangent track. Truly speaking, if we measure any curve in service, there will always be some variation in the versines from station to station and therefore, the simple curve will have some variations in radius at various points on the curve. Unless the radius is designed to be different in different parts, such slightly disturbed curve will still be called a simple curve.

![Simple Curve Diagram](image)

Fig. 1.16: Simple Curve

1.10.2 Compound Curve: A compound curve is a combination of two or more circular curves of different radii in similar flexure (i.e. having same direction of radius from center). (see figure 1.17) Between the curves of different radii, transition curve may be provided. Two curves of similar flexure separated by a small tangent or straight length in between are two separate curves and not a compound curve. Normally, compound curves are introduced in the alignment in the following circumstances:
Fig. 1.17: Compound Curve

a) Difficult layout: When the track alignment is to pass through a restricted space due to multiple obstructions on either side of the proposed track, the simple curve might not be feasible and compound curves are the only alternative. (see figure 1.18)

b) Where the length of transition is not available at one end, the larger radius curve may be introduced at that end, and compounding may be done to get the full speed potential on the curve.

Fig. 1.18: Compound curve (R₁ and R₂ are in same direction) set out in a very congested area between a water body and built up area.

Note: The extended curve 1 is cutting through the built up area and if only curve 2 is set, it will not be tangential to tangent 1.
c) During service, when the curve gets disturbed and it becomes too costly, difficult or non-feasible to get back to the original alignment, an easier realignment solution may be obtained by compounding the curve slightly.

1.10.3 Reverse Curve: A reverse curve is a combination of two circular curves of contrary flexure (i.e. having opposite direction of radius from the center). (Figure 1.19) Two curves of opposite flexure, separated by small tangent or straight length in between will still be called a reverse curve. Reverse curves are introduced in track when the track alignment is to be shifted laterally, by and large parallel to the original direction. Such shifting is required at difficult layout locations where there is some obstruction in the form of any structure, water body hill etc.

Fig. 1.19

Where we need extra space between tracks such as for construction of structures such as bridges, tunnels, platforms etc. temporary or permanent diversions may be laid for the above purpose and for this purpose we introduce a pair of reverse curves into the track. (Figure 1.20).

Fig 1.20
1.10.4 Minimum Straight between reverse curves: On high speed routes, a minimum length of straight is required to be provided between curves of contrary flexure. This straight is required so that the vibrations induced in the vehicle at the end of one curve die out before the vehicle enters the next curve. In the absence of adequate straight length, the oscillations of the first curve will interfere with the oscillations of the second curve and the passengers will be discomforted. This requirement is especially important at higher speeds.

The desirable length of straight between two reverse curves is 50m on BG and 30m on MG. If this minimum length cannot be provided, the two curves shall be so extended as to eliminate the straight in between. The rate of change of cant and versine along the two transitions shall be kept the same so that the vehicle moves over curves with the same period of roll. If neither of the two is possible a speed restriction of 130kmph on BG and 100 kmph on MG shall be imposed.

Note for Maintenance: The above mentioned provisions of IRPWM are applicable for high speed trains only. However, during maintenance the versines of the curve keep on getting shifted beyond the original curve length. Therefore if there is no straight between reverse curves, it will be very difficult to maintain the junction of the two curves. Therefore, as a general good practice it is desirable to have some straight length between reverse curves.

1.10.5 Length of transition in case of compound and reverse curves. The transition curve is to be provided to take care of the difference in the curvatures and superellevations of the two curves joined by the transition. In case of compound curves, the two curves have the same direction of the curvature. Therefore, the desirable length of transition, L in m, between the compound curves shall be more than the maximum of following three values:

\[ L = 0.008 \times (C_{a1} - C_{a2}) \times V_m \] \hspace{1cm} (1.15)

\[ L = 0.008 \times (C_{a1} - C_{d2}) \times V_m \] \hspace{1cm} (1.16)

\[ \text{Para 407 (8) of IRPWM.} \]

\[ \text{Para 407 (9)(b) of IRPWM.} \]
If there is no straight between reverse curves, the two curves joining at the transition have curvature in different directions. Therefore, the desirable length of transition, L, between the reverse curves shall be more than the maximum of the following three values:

a) \[ L = 0.008 \times (C_{a1} + C_{a2}) \times V_m \]  
   \( 1.18 \)

b) \[ L = 0.008 \times (C_{d1} + C_{d2}) \times V_m \]  
   \( 1.19 \)

c) \[ L = 0.72 \times (C_{a1} + C_{a2}) \]  
   \( 1.20 \)

Where \( C_{a1} \) and \( C_{a2} \) refer to the actual cant in mm for curve no 1 and curve no 2, \( C_{d1} \) and \( C_{d2} \) refer to cant deficiency in mm for curve no 1 and 2 which form the compound/ reverse curve.

and \( V_m \) = Maximum permissible speed on the curve (KMPH)

On BG, the above length can be reduced in exceptional circumstances in similar fashion as for simple curves, as given in para 1.8.8.

**Note:** Where there is a straight track between two curves (reverse or compound), the curves are independent of each other and the formulae for calculating the lengths of transitions for simple curve will be applicable.

1.10.6 Versine and Cant Diagrams for Reverse/ Compound Curve: As per the above design, the versine/ cant diagram for the reverse and compound curves shall be as follows: (Figure 1.21 and 1.22)

---

Para 407 (9)(a) of IRPWM.
1.11 Shift: During the final location survey, if it is decided that a curve is to be provided, first of all the apex point is fixed up from the two tangents decided on the basis of change in direction desired. Then we first decide the circular curve of suitable radius which can be provided, as discussed in para 1.6.1 (Figure 1.7). After this, the desired transition lengths are introduced between the straight and the circular portion of the curve. Figure 1.21 shows the two transition curves inserted at either end of the curve. Each transition is inserted half in the straight and half in the curved track. In order that the transition curve meets the circular curve tangentially, the circular curve T-T’ is required to be shifted inside to TC-CT. (The circular curve T-T’ is called as equivalent circular curve and this concept is used when we carry out the realignment of curves in chapter V.)
This shifting of the circular curve inwards to meet the transitions is called shift. Shift is a geometrical property of a curve and cannot be avoided if the transition curve is to be provided. If there are constraints to shifting of curve inwards due to physical features such as permanent structures, water bodies, yard layouts, private land etc. the full amount of transition desired cannot be provided. To calculate the shift, S, let us draw a part of Figure 1.23 showing only one tangent and a part of curve. The tangent is drawn horizontally in the Figure 1.24.

![Figure 1.24](image)

The circular curve originally chosen is starting at F. Due to introduction of the transition curve, AB, the ordinate at B is BG. The circular curve shifts inwards to meet the transition curve at B.

In order that the transition curve and circular curve meet, the circular curve is shifted inwards by an amount EF. (It may be seen that shift is measured between the original circular curve without transition and the circular curve after the transition has been provided, and not between the straight and the circular curve i.e. shift is EF and not BG.)

Now since \( EF = DF - DE = BG - DE \)

Using equation (1.11) for the transition, \( y = \frac{x^3}{6RL} \)
\[ BG = y_{(x=L)} = \frac{L^3}{6RL} = \frac{L^2}{6R} \]

For the arc BEC of the circular curve, BC is the chord with Length equal to the length of transition, \( L \),

\[ \therefore \text{Using eqn. (1.2) DE is the versine for the chord i.e. } \quad DE = \frac{L^2}{8R} \]

\[ \therefore EF = BG - DE = \frac{L^2}{6R} - \frac{L^2}{8R} \]

i.e Shift = \( \frac{L^2}{24R} \) ........................(1.21).

If we take the length of transition in metres, radius in metres and shift in centimeters, the above equation reduces to\(^{23} \)

\[ \text{Shift, } S = \frac{4.2 \times L^2}{R} \]

Considering \( F\ H = Y_{(x=L/2)} = \frac{(L/2)^3}{6RL} = \frac{L^2}{48R} \) .............(1.22)

i.e. The offset in the transition curve at point \( F \) (where equivalent circular curve starts from straight is equal to half the shift of circular curve.

The value of shift depends on:

i) Radius, \( R \)

ii) Length of transition, \( L \)

If there is problem in laying curve due to problem of shift, radius of curve or length of transition are to be manipulated so that the curve can be laid following the constraints at site. If neither is possible, the length of transition is to be limited and this will reduce the speed potential of the curve and a permanent speed restriction has to be imposed.

1.11.1 Curves without transition: There may be certain layouts in difficult terrain or yards where it is not possible to accommodate

\(^{23}\) Para 407(5)(b) of IRPWM.
any shift. In such layouts, changing the radius is also generally not possible. Such curves may have to be laid without transition. In such a case, the vehicle enters the curve without any transition and a jerk will be felt due to sudden introduction of curvature and lateral acceleration. This will lead to severe discomfort to the passengers in the vehicle. However if we take the length of transition as zero, the speed potential of the transition curve will also be zero using the formulae given above.

But is it possible to have a curve without any transition in reality? Let us examine this question:

If we carefully see the motion of the vehicle on the curve (see figure 1.25), the vehicle as a whole cannot be suddenly shifted
from the straight to the curve. Only one wheel of a vehicle or bogie enters the curve at a time and the other one remains on the straight for some time depending on the speed of the vehicle. The actual turning of the vehicle into curve occurs over the length of bogie and not instantaneously as presumed with zero transition length.

In figure 1.25, there are three stages:

Stage I: The first wheel is about to enter the curve (it is at the junction of straight and circular curve), but both the wheels are still in straight track. In this situation, the versine experienced by the vehicle is zero.

Stage II: The first wheel has entered in curve whereas the second wheel is on the straight track. In this situation, the vehicle is having some versine, which is non zero but the same is less than the situation when the complete vehicle is in curve.

Stage III: The second wheel is about to enter the curve, (second wheel is on junction of straight with circular curve) and both the wheels are now in curved track. In this situation, the versine is equal to the versine in the circular curve.

As is evident from the versine diagram drawn in figure 1.25, between the time where the first wheel enters the curve (stage I) and the second wheel enters the curve is stage III, the bogie or body turns slowly, as if on a transition. The versine diagram drawn at bottom of figure 1.25 also shows transition as seen in figure 1.15. The bogie/ wheel base of the vehicle ‘virtually’ provides a transition from straight to the curve. **This length over which this turning of the vehicle occurs is called virtual transition.**

Since the comfort is the main criteria in this case, the virtual transition is seen between the front and rear ‘bogie center’ instead of the ‘wheels’ as the passengers experience the jerk between the center pivots of the vehicle and not from the wheels themselves. The values of virtual transition laid down in the IRPWM are based on the distance between bogie centers for passenger trains. These values are laid down as 14.6 m for BG, 13.7 m for MG and 10.3 m for NG.24

The role of virtual transition is that the cant is introduced/run-out on this length subject to the maximum cant gradient allowed. As in case with other transitions, the virtual

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24 Para 405(2) (a) of IRPWM
transition commences at half its length before the tangent point of the curve on the straight and terminates at the same distance beyond the tangent point on the curve.

**Note for Maintenance:** In case of virtual transitions, the front wheels suddenly enter and leave the curve and these experience a severe jerk while entering or exiting from the curve. This jerk is also transferred to the vehicle. From the passenger comfort consideration, provision of such a non-transitioned curve is not recommended and may lead to discomfort and maintenance problems. It is prudent from maintenance point of view that an actual transition of the length equal to virtual transition may be provided. It may be seen that on a sharp 600m radius curve on BG, if we provide a small 14.6m transition, there will be a shift of only 14.8mm which can be easily provided.

### 1.12 Vertical Curves

The curves discussed so far are all horizontal curves and these curves facilitate change in alignment of track in horizontal plane. Another category of curves is possible: when the track changes direction in the vertical plane (i.e. gradient changes). When a vehicle moves from a steeper up gradient to a less steep up gradient (or less steep down gradient to steeper down gradient), the junction is called **summit** type vertical curve. When a vehicle moves from a less steep up gradient to a steeper up gradient (or steeper down gradient to a less steep down gradient), the junction is called **sag** type vertical curve (figure 1.26).

![Vertical Curves Diagram](image)
Just as in the case of horizontal curves, when a vehicle is moving on vertical curve, centrifugal force is acting on the vehicle acts away from the center of the vertical curve. These forces act in addition to the weight of the vehicle and have to be managed to avoid undesirable effects on the vehicle. This effect is similar to the experience of people moving in a lift. When the lift moves down (similar to sag type vertical curve) there is on-loading and when the lift moves up (similar to summit type vertical curve) there is off-loading.

The following considerations are there in providing the vertical curves:

1.12.1 Safety:

a) Summit type vertical curves: The centrifugal force acts upwards in case of vehicle moving on summit type vertical curves. This is opposite to the weight of the vehicle and causes off-loading (reduction in wheel load) of the vehicles. The offloading of wheels if coupled with lateral forces may lead to derailment of vehicle on track. Therefore, while designing the vertical curves, the radius is to be chosen such that this offloading is within safe limits.

b) Sag type vertical curves: The centrifugal force acts downwards in case of vehicles moving on sag type vertical curves. This acts in the same direction as the weight and normally does not have any harmful effect on the vehicle except slightly higher load on springs and axles. However, if the train is on sag type vertical curve and brake is applied from locomotive, the front portion near locomotive will start decelerating, whereas the rear portion, which is on a down gradient, will continue to move forward due to the effect of gravity. In such a case, the vehicles may bunch together and if the vehicles on the sag are empty, these may get lifted up, causing off-loading. In this case, the vehicles may be prone to derailment. Properly designed vertical curves will reduce the chances of this off loading.

1.12.2 Passenger Discomfort: As discussed above, in case of vertical curves of summit type, off-loading is there and in case of sag type vertical curves, on-loading is there. In either of the two cases, passengers will be discomforted if the
acceleration is high. The limits of vertical acceleration are generally accepted as 0.3 to 0.45m/sec$^2$.

The radius of the vertical curve can be worked out based on the following relationship between speed of the vehicles, radius of the vertical curve and permissible values of vertical accelerations. $R_v \geq \frac{V_m^2}{a_m}$

Where, $R_v$ = Radius of vertical curve in meter  
$V_m$ = maximum permissible speed of the vehicle in m/sec  
$a_m$ = permissible vertical acceleration in m/sec$^2$

To keep the vertical acceleration within acceptable limits, properly designed vertical curve is necessary to be provided. However, these curves are required to be provided only where the difference in the gradients meeting is large and the forces generated may lead to unsafe conditions/discomfort. The vertical curves provided on IR are circular. In vertical curves, there is no requirement of transitions, since there is no superelevation and also since the forces generated are small due to the very large radii of the vertical curves. The vertical curves are to be provided only when the algebraic difference in gradients meeting at a point is equal to or more than 4 mm per metre or 0.4 percent$^{25}$.

The guidelines for provision of vertical curves are$^{26}$:

<table>
<thead>
<tr>
<th>Classifications of Routes</th>
<th>Minimum Radius (metre)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Group A – BG</td>
<td>4000</td>
</tr>
<tr>
<td>Group B – BG</td>
<td>3000</td>
</tr>
<tr>
<td>Group C, D, E – BG &amp; MG</td>
<td>2500</td>
</tr>
</tbody>
</table>

**Note for maintenance:** The vertical curves of the summit type can only be provided in the formation during earthwork. It is very difficult to provide the same in service. Extra care shall be taken during construction/gauge conversion of a line to provide proper summit type vertical curves. As regards the sag type vertical curves, some improvement can be made in service by increasing

$^{26}$ Para 419 of IRPWM.  
$^{27}$ Para 419 of IRPWM.
the ballast cushion. But it will be good from maintenance perspective if properly designed vertical curves are provided in the formation during the construction of new lines/ gauge conversion.

1.13 CHAPTER I REVISION QUESTIONS

1. What is a curve? Why curves are required in alignment?
2. How are the curves designated and measured?
3. What are lateral forces acting in case of a train moving on curved track?
4. What are disadvantages of providing high cant?
5. Discuss the factors which govern maximum cant deficiency and cant excess.
6. What is equilibrium speed? How is it decided?
7. Why transition curves are required at the ends of the curves?
8. What is shift and how inability to provide required shift affects the train operations?
9. What is virtual transition?
10. Where do we provide vertical curves? Discuss the effects of vertical curves on maintenance and safety.
CHAPTER II

MAINTAINING CURVES
MAINTAINING CURVES

2.1 Inspection of Curved Track: On Group 'A' and B' routes, gauge, versines and superelevation on each curve must be checked once in every four months and on other routes every six months. Such checks should also be carried out whenever the running over curves is found to be unsatisfactory. The versines, superelevation and gauge should be recorded by the Permanent Way Inspector (JE/SSE/P.Way) in the curve register as per the proforma given as Annexure-4/5 of IRPWM. The Assistant Divisional Engineer shall check at least one curve of each Permanent Way Inspector (SSE/P.Way - Incharge) every quarter by taking its versine and superelevation as well as gauge from end to end. The decision to realign should be taken by the Permanent Way Inspector-in-charge(SSE/P.Way - Incharge) or Assistant Divisional Engineer. The realignment of curve should be carried out in dry season and not during rainy season except when this is unavoidable.

Note: These schedules are for the horizontal curves. No schedules are laid down for the vertical curves.

2.2 Measurement and Record of a Curve: The curves shall be measured and the record of the same shall be kept in a register as per the proforma given in IRPWM. The curves in each SSE section shall be numbered serially from one end in direction of increasing kilometers i.e. curve nos 1, 2, 3 etc. On multiple line sections, the curves are identified with suffix

\[1\] Para 420 (2) of IRPWM
indicating the line as curve nos 15 DN, 7 UP or 8 Line no. 3 etc. If additional curves are inserted due to some diversion etc, the curves may be numbered with a, b etc as suffix. For example, if a reverse curve is inserted between curve nos 8 DN and 9 DN, the two new curves can be numbered as 8a DN and 8b DN. The measurement of the curve includes the measurement of gauge, cross levels and versines. The gauge and cross levels are to be measured using the gauge-cum-level at each station in the same manner as for a straight track. As already discussed in chapter I, the versine is the parameter used to measure the curvature of the curve. Following chord lengths are normally used on IR:

- For curves in normal track, 20m chord length is used on stations 10m apart.
- For lead portion of turnouts and turn-in-curves, 6m chord length is used on stations 3m apart. (Para 237 (4) (C))

### 2.2.1 How to Mark station numbers:

The stations at which the readings for measurement are to be taken are nominated and are paint marked on the track. The station numbers shall be marked starting from a few stations ahead of the actual start of the curve and upto a few stations beyond the last station of the curve. As per practice, the station numbering shall commence from zero on the first station on the curve and shall increase in the direction of increasing kilometers. The stations ahead of the curve shall be marked -1, -2, -3 etc from the 0 station backwards and the stations beyond the curve shall be marked as +1, +2, +3 etc from the last station onwards.(Figure 2.1)

### 2.2.2 Method of taking measurements:

In case of curves in normal track, the 20 m long chord is taken between three stations, each 10 m apart, and the reading on scale at the middle station is called the versine at that station (figure 2.2). In actual site, a fishing chord is stretched between three stations. The chord is held 13 mm below the rail table for measuring the versines. The chord is stretched on the gauge face of the outer rail. For the versines in turnouts/ turn in curves, 6 m long fishing chord is stretched between three stations, each 3 m apart and the reading taken on the middle station is
Fig. 2.1: Station no painted on web of rail

called versine for that station. Wooden cubes of size 20mm/ 25mm are used, and the size of wooden block is deducted from the reading taken on scale. (figure 2.3)
The readings shall be taken to nearest millimeter and the fishing chord shall be properly tightened so that errors due to slack chord are not there in the readings. After taking one reading, chord is shifted by half chord length at a time to get versine readings at all stations on the curve.

**Standard Tool Kit for P Way Officials:** In the standard tool kit for the P-Way officials, the curve measuring equipment is provided. In this case, steel chord holders are used (figure 2.4) for stretching the chord which holds the chord 25 mm away and 13 mm below rail table. In the tool kit, special scale is provided (figure 2.5) which is marked after deducting the distance of chord from gauge face. This method is very convenient and the reading in the special scale is directly noted. Since there are no manual calculations for the deduction of the distance of the chord from the gauge face, this avoids mistakes during calculations.

The positive values of versines indicate curvature in one direction, and negative values of versines indicate curvature in other direction. When measuring reverse curves, the versines are taken on the outer rail in one curve. Near the junction point, the rail for taking readings is required to be changed but care is to be the entries are made with appropriate sign to indicate curvature in opposite direction.
2.2.3 Where to Mark station numbers: The stations for taking measurements shall be marked on the inside face of the web of the outer rail. The super-elevation in the transition portions is to be indicated on the inside face of the web of the inner rail opposite to the station. In circular portion, super-elevation shall be indicated at its beginning and end on the inside face of the web of outer rail. For long circular curves, super-elevation value shall be indicated at intermediate stations at distance not exceeding 250m.

2.2.4 Record of curve inspection: The record of the measurement of curve shall be kept in curve register as per the proforma given in IRPWM. The proforma specified are as follows:
### PROFORMA FOR CURVE REGISTER

#### ABSTRACT OF CURVE

<table>
<thead>
<tr>
<th>Sl. No. (1)</th>
<th>Curve No. (2)</th>
<th>Between Stations (3)</th>
<th>Kilometers From (4)</th>
<th>To (5)</th>
<th>R.H. or L.H. (6)</th>
<th>Degree of Curve (7)</th>
<th>Radius of Curve (8)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>S.E. (9)</th>
<th>Length of Transition (10)</th>
<th>Circular Curve (11)</th>
<th>Total Length of Curve (12)</th>
<th>Whether Joints are Square or Staggered (13)</th>
<th>Whether Reference Pillars have been Provided (14)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### PROFORMA FOR CURVE REGISTER

#### DETAILS OF INSPECTION

<table>
<thead>
<tr>
<th>Railway</th>
<th>Curve No.</th>
<th>Degree of Curve</th>
<th>From KM</th>
<th>To KM</th>
<th>Section</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Station No (1)</th>
<th>Prescribed (Ideal)</th>
<th>Date of Check (5)</th>
<th>Measurements Recorded</th>
<th>Action to be taken</th>
<th>Date of String lining or Local Adjustment (10)</th>
<th>Measurements recorded after adjustment with date of check</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>V (2)</td>
<td></td>
<td>V (6)</td>
<td></td>
<td>V (11)</td>
<td>V (11)</td>
</tr>
<tr>
<td></td>
<td>SE (3)</td>
<td></td>
<td>SE (7)</td>
<td></td>
<td>SE (12)</td>
<td>SE (12)</td>
</tr>
<tr>
<td></td>
<td>G (4)</td>
<td></td>
<td>G (8)</td>
<td></td>
<td>G (13)</td>
<td>G (13)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**2.3 Ballast in curves:** During the passage of trains over curves, the wheel takes guidance from the outer rail. Further, the centrifugal / centripetal forces are acting. In addition to these lateral forces acting on the vehicle as well as track, the continuation of SWR/LWR on curves creates extra lateral forces acting on the track and vehicles, which can lead to increased wear and tear. Proper maintenance and inspection are crucial to ensure safe and efficient operation of the railway system.

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2 Annexure 4/5 of IRPWM
forces due to the thermal effect. The track geometry can be retained in such situation only through adequate lateral support through extra ballast in shoulders of the track. LWR/SWR are very much susceptible to buckling on the curved track as any misalignment, coupled with already present lateral forces will assist buckling, if the track is not properly restrained from sides. Therefore the ballast shoulder width required on curves is more than that required in straight track due to the extra forces aforementioned. Further, ballast cushion is required in a particular shape to maintain the superelevation and elasticity of track. Therefore, good ballast cushion and shoulders are necessary to ensure proper performance of track on curves.

2.3.1 Specified shoulder width for curves in BG

a) Fish Plated Track:\(^3\)
   - For straight track and curves flatter than 600m radius: 305 mm
   - For outer side of curves sharper than 600m radius: 400 mm
   - For turn in curves of turnouts in passenger yards: 550 mm on outside of curves.

b) Short Welded Rail track:\(^4\)
   - For straight track: 305 mm
   - For outer side of curves up to 875m radius: 400 mm
   - For outer side of curves sharper than 875m radius: 450 mm
   - For turn in curves of turnouts in passenger yards: 550 mm on outside of curves.

c) Long Welded Rail Track:\(^5\)
   - For straight track: 350 mm
   - For outer side of curved track: 500 mm
   - For outer side of curves sharper than 440m radius and up to 360m radius in zone-1: 600 mm\(^6\)
   - For reverse curves sharper than 1500m radius: 600 mm over length of 100m on either side of the common point

\(^3\) Annexure 2/13 Para 263 IRPWM
\(^4\) Para 263 of IRPWM (Annexures 2/11, 2/13)
\(^5\) Annexure 2/11 Para 263 IRPWM, Para 3.2.2 LWRM
\(^6\) Para 3.2.1(ii) of LWR Manual corrected as per CS No 11
The extra shoulder width shall be provided for 100m beyond the tangent point.\(^7\)

2.4 Earthwork for curves: The earth work for the curved alignment shall be done after all the parameters of the curve have been identified and frozen. The alignment including shift shall be properly laid out before the earthwork is commenced. The width of the formation in curves is more than that for straight alignment if extra ballast is provided in shoulders, and the width is to be increased further for the extra clearances as discussed in para 2.13 of this chapter.

While carrying out the earthwork, there are two options regarding the cross slope in the earthwork top:

a) The formation may be laid similar to the that of straight, having minimum 1 in 30 slope\(^{12}\) in either direction from the center line to cater to the drainage requirements. This method will be suitable where the degree of curve and the cant provided are low. In this method, the cant will be provided by differential depth of ballast cushion under the inner rail seat and outer rail seat.

b) On highly canted track, if the above method is followed, the ballast requirement will go up considerably and the maintenance of track in service will also be difficult due to the differential depth of ballast under the inner/outer rails. Therefore, for highly canted track, it is desirable that the cross slope equivalent to the super-elevation be provided in the formation. The two-directional formation slope in straight shall be changed by introducing a transition portion and changed to one-directional slope required on the curved alignment. Separate measures such as the hidden drains/pipes etc are required to be taken for ensuring drainage in the transition portions of reverse curves. Otherwise, the most vulnerable part of the curve, i.e. the transition portion in reverse curve will have drainage problems also.

Note: 1) The laid down instructions allow only the option a) at the moment and do not recognise option b)

2) In case of working with BCM, the slope is provided in one direction only and the option b) only is available regarding the slope in formation for all types of track.

\(^{7}\) Para 3.2.1(ii) and 3.2.2 of IR LWR manual
2.4.1 Formation Width for Curved Track: Extra width of formation is required to be provided on curves to accommodate the extra ballast shoulder widths as given in para 2.3.1 above.

2.5 Providing Superelevation in Curves: Superelevation is to be provided in the curves as per design. It is normally constant in circular curve and varying in transition portion. The variation of superelevation may be done as follows:

Running out Superelevation

(1) On transitioned curves, cant should be run up or run out on the transition, not on the straight or on the circular curve, increasing or decreasing uniformly throughout its length.

(2) On non-transitioned curves, cant should be run up or run out on the ‘virtual transition’ in a length of 14.6 m half in straight, half in circular curve.

(3) Longitudinal profile of transition on the reverse curve may be in one of the following two alternatives shown in figure 2.6.

In case no. 2, the level of the centre line of the track is maintained the same throughout, and the cant is provided by raising one rail by half the amount of cant and lowering the other rail by the equal amount. Cant is run out or gained over the length of the transition by raising and lowering both the rails by equal amount symmetrically, with respect to the level of the centre line track. In case of no. 1, the level of the centre of the track gets disturbed whereas in case of No. 2, it is maintained the same throughout.

(4) Special cases of superelevation run out may be approved by the Chief Engineer.

Note for maintenance: If in service the superelevation is not found as per the design and is required to be adjusted, track is generally not lowered but inner or outer rail is raised, as required.

2.6 Gauge in Curves: The minimum clearance required to permit the vehicles to negotiate a curve with free curving motion depends on the maximum rigid wheel base and the degree of curve for a particular track gauge. Over and above the minimum clearance, it is desirable to have 3 to 5 mm additional clearance to cater to the angularity assumed.

Para 408(3) IRPWM
by the wheel set while negotiating the curve. Gauge widening becomes necessary when the requirement of the clearance exceeds the standard play. The required gauge widening and the limits with which it should be laid/ maintained are given below:

<table>
<thead>
<tr>
<th>Gauge</th>
<th>Radius of Curve</th>
<th>Amount of gauge widening during maintenance w.r.t. nominal gauge</th>
<th>Gauge to be laid within</th>
</tr>
</thead>
<tbody>
<tr>
<td>B.G.</td>
<td>( \geq 440 \text{m} )</td>
<td>-6mm to 15mm</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>( \geq 350 \text{m} )</td>
<td>-</td>
<td>-5mm to +3mm (1671 to 1679mm)</td>
</tr>
<tr>
<td></td>
<td>&lt;440m</td>
<td>Upto + 20mm</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>&lt;350m</td>
<td>-</td>
<td>Upto + 10mm (Upto 1686mm)</td>
</tr>
<tr>
<td>M.G.</td>
<td>( \geq 290 \text{m} )</td>
<td>-3mm to +15mm</td>
<td>-2mm to +3mm</td>
</tr>
<tr>
<td></td>
<td>&lt;290m</td>
<td>Upto + 20mm</td>
<td>Upto + 10mm</td>
</tr>
<tr>
<td>N.G.</td>
<td>( \geq 400 \text{m} )</td>
<td>See note below</td>
<td>-3mm to +3mm</td>
</tr>
<tr>
<td></td>
<td>&lt; 400m</td>
<td></td>
<td>Upto + 10mm</td>
</tr>
<tr>
<td></td>
<td>Upto 100m</td>
<td></td>
<td>Upto + 20mm</td>
</tr>
<tr>
<td></td>
<td>&lt; 100m</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: Amount of gauge widening during maintenance with respect to nominal gauge of 1676 mm can be as follows:

(i) On curves with radius of more than 175 metres: 3 to +15mm

(ii) On curves with radius less than 175mm: upto +20mm
2.6.1 Providing wide gauge sleepers: The normal PRC sleepers used in the straight have nominal gauge of 1673 mm. Where the wider gauge is to be provided, special sleepers having wider nominal gauge shall be provided. The extra widening of the gauge has to be introduced in the transition portion of the curve, and sleepers are also to be provided accordingly. The gauge shall be changed from normal to the wider gauge in curve at the rate of $1/3$rd widening in each $1/3$rd length of transition\(^9\), as shown in the figure 2.7.

![Track Layout](image)

**Fig. 2.7**

2.6.2 Laying Sleepers in curves: On curved track, the sleepers are required to be laid radial to the curve. To ensure this, the spacing should be correctly marked on outer rail and then the position can be transferred on the inner rail with the help of a ‘T’ square.

2.6.3 Permitted Wear In rails on curves: The wear shall be measured on curved track during inspections. As per IRPWM para 428, for curves sharper than 600 m on B.G. and 300 m on M.G., regular wear readings shall be taken. The measurement shall be done using templates (Figure 2.8 shows measurement of lateral wear using template).

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\(^9\) Para 403 (1), 224(2)(e)(v) IRPWM, Para 19, Chapter 1 of BG SOD

\(^{10}\) RDSO drawing no RDSO/T-4183 to RDSO/T-4186
Fig. 2.8: Measurement of wear on outer rail

a) Maximum Permitted Lateral wear\textsuperscript{11}

<table>
<thead>
<tr>
<th>Section</th>
<th>Gauge</th>
<th>Category to track</th>
<th>Lateral wear</th>
</tr>
</thead>
<tbody>
<tr>
<td>Curves</td>
<td>B.G.</td>
<td>Group ‘A’ &amp; ‘B’ Routes</td>
<td>8mm</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Group ‘C’ &amp; ‘D’ Routes</td>
<td>10mm</td>
</tr>
<tr>
<td></td>
<td>M.G.</td>
<td>Group ‘Q’ &amp; ‘R’ Routes</td>
<td>9mm</td>
</tr>
</tbody>
</table>

b) Maximum Permitted Vertical Wear: \textsuperscript{12}

<table>
<thead>
<tr>
<th>Section</th>
<th>Gauge</th>
<th>Rail Section</th>
<th>Vertical wear</th>
</tr>
</thead>
<tbody>
<tr>
<td>Curves</td>
<td>B.G.</td>
<td>60 KG/M</td>
<td>13mm</td>
</tr>
<tr>
<td></td>
<td></td>
<td>52 KG/M</td>
<td>8mm</td>
</tr>
<tr>
<td></td>
<td></td>
<td>90 R</td>
<td>5mm</td>
</tr>
<tr>
<td></td>
<td>M.G.</td>
<td>75 R</td>
<td>4.5mm</td>
</tr>
<tr>
<td></td>
<td></td>
<td>60 R</td>
<td>3mm</td>
</tr>
</tbody>
</table>

2.6.4 Reducing Wear on Outer Rail of Curves: \textsuperscript{13} Since the vehicle takes guidance from the outer rail, and the wheelbase is rigid, lot of side wear is there in the outer rail. To reduce the wear, the following actions shall be taken:

a) Lubrication of gauge face of outer rails: The lubrication of the gauge face of the outer rail (taking care not to lubricate the rail top, which can lead to wheel slips) on the curves,

\textsuperscript{11} Para 302(1) (iv) IRPWM
\textsuperscript{12} Para 302(iii) of IRPWM
\textsuperscript{13} Para 427 and 250(3) of IRPWM
Fig. 2.9: Lubricated outer rail of curve

Fig. 2.10: Automatic curve lubrication
either manually or using Rail Flange Lubricators will reduce the friction and thence the wear (Figure 2.9). In curves where the wheel flange touches the outer rail, lubrication shall invariably be done.

Rail flange lubricators should be provided on curves of radius 600 meters and less on Broad Gauge and of radius 300 meters and less at Meter Gauge to avoid rail face wear, the first lubricator being provided a little ahead of the curve. (Figure 2.10). Otherwise lubrication may be done by keyman/trackman at regular intervals.

b) Maintain correct curve geometry and superelevation in curve: The correct gauge, versine and superelevation will ensure smooth movement of the trains, aid gradual introduction / removal of the lateral forces and will keep the lateral forces within the limits. This, in turn, will reduce the wear on the rails and fittings.

c) Choosing proper cant: Choosing cant to corresponding equilibrium speed shall ensure the wear to be equal in both

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\[14\] Para 4, Chapter I, BG SOD
the rails. Otherwise if cant is too low outer rail will get wornout too fast and if cant is too high, inner rail will get flattened too fast.

d) Provision of the suitable check rail: On sharp curves, to reduce lateral and angular wear on outer rail, as well as to increase stiffness of track to prevent distortion of track due to lateral forces, check rails are provided. The check rails shall be provided in curves sharper than 8 degree (Radius sharper than 218m) or in flatter curves if higher speeds are contemplated. (Figure 2.11)

Notes for maintenance:

1) The check rails are normally assumed to be sharing the load with the outer rail in wheel guidance. However, as explained in the para 7.3.1 chapter 7 (Track Defects) in the book “The Investigation of Derailments” by Sh R K Yadav, published by IRICEN, the action of the check rail and the outer rail in guiding the wheels are not simultaneous and only one of the two guides at a time, depending on the thickness of the wheel flange.

2) The check rail is not a panacea for all the ills of track maintenance on the curves. After providing the check rail, the maintenance of track requires more efforts since

- Track structure gets heavier due to the check rail and requires extra efforts during maintenance for lifting/slewing etc.
- Providing and opening of the fittings of inner rail such as ERCs on the inside and fish plates etc becomes more cumbersome due to the provision of check rail,
- Attention to joints is difficult as the space for packing gets constrained due to check rail (especially when the check rail joint is also nearby),
- During tamping, the check rail is required to be opened out which adds to maintenance efforts.

It may be a good idea to bring UNIMAT machine for tamping of curves with check rails so that the problems posed in opening/refixing of check rails are avoided as also the proper tamping of joint sleepers is ensured.
Further, during tamping, the sleepers invariably get disturbed and re-fixing of the check rail becomes problematic.\textsuperscript{15}

- Maintenance of the clearance of check rail is difficult due to the heavy load from the wheels. If clearance is slack, the check rail becomes ineffective
- Fittings of check rail require frequent tightening and renewals.

3) However, the check rail does help in improve stiffness of track and physically holds the inner wheel if outer wheel starts to derail. Therefore, a well maintained check rail with good fittings will certainly help promote the safety on sharper curves.

4) Clearance in Check Rail: The minimum check rail clearance shall be 44 mm. This value shall be increased by half the amount of difference between 1676 mm and the gauge to which the curve is actually laid. i.e. Clearance shall be\textsuperscript{16} \( = 44 + (G_c - 1676)/2 \), where \( G_c \) is the design gauge in the curve.

2.6.5 Interchanging of Rails: Despite all the efforts, the wear on the rail in curved track is more than that in the straight track. Increased life from the same set of rails can be obtained by interchanging the rails when wear reaches the permissible limit. By doing this, we change the wear location from the side of the head in outer rail to the rail top in inner rail, and vice versa. However, at the time of turning, matching of rail ends on the gauge face should be ensured.

2.7 Maintaining safety on curves: When the train negotiates a curve, various conditions favourable for derailment are there:

a) Positive angularity is present: The wheel takes guidance from the outer rail and curving forces cause the wheelset to take positive angularity which is favourable to derailment process.

b) Lateral forces are present: In curves, lateral force (residual of centrifugal and centripetal forces) is almost always present. When the vehicle is traveling at speed higher than

\textsuperscript{16} Para 4, Chapter I, BG SOD
equilibrium speed, the lateral force acts outwards, which is favourable to derailment process.

c) **Offloading of wheels is there:** When the vehicle moves at speed less than equilibrium speed, the reaction on inner rail is more than that on outer rail. This results in offloading of outer wheel.

d) **Design twist is there:** In the transition portion of curve, design twist is there for introducing/removing the superelevation in the circular portion of curve.

The above factors eat into the safety margin available on straight track against the safety. Any further defects in track parameters, vehicle defects and engine-manship might easily result in unsafe conditions for the vehicles. Therefore, extra precautions are required in track maintenance on curves. Lubrication of gauge face of outer rail and provision of check rails in sharp curves are important means of ensuring safety on the curves.

### 2.8 Carrying Out Mechanized Track Maintenance (Tamping) on Curves:

Because of the lateral forces exerted by railway vehicle on track, there is more possibility of track going out of alignment on curve compared with the straight track. In any case, the packing of the sleepers gets loose with the passage of traffic over it. So there is a need to correct the track parameters of curve and pack the sleepers regularly. With track structure getting heavier, the attention of curve, whether as regular maintenance activity or as complete/partial realignment of curve, is done using ontrack machines such as DUOMATIC, CSM etc. by tamping. The tamping may be done in two modes:

a) **Smoothening mode** i.e. automatic correction of the geometry as per the features of the curve. This mode is chosen when the existing geometry of the curve is satisfactory. The tamping in smoothening mode will reduce the defects in geometry somewhat but will not completely eliminate them.

b) **Design mode** is chosen when it is desired to bring the track to the desired geometry. In this method, the existing curve is measured, the desired geometry solution is found out either manually or using computers, as given in chapter V of this book.
While tamping the curved track by machines, there are certain features of the machine working which should be properly understood before we go for tamping of curve. While packing of curve by machine, 4-point method of lining or 3-point method of lining by machine are used.

2.8.1 4 point lining system: In this case, the 4th trolley which is normally kept inside the machine is opened out and the chord length increases. Four points are now available on the curve as shown in figure 2.12. The machine is working from point A towards point D. Point A and B have already been attended and the point C is being attended.

Trollies at A, B, C and D are pneumatically pressed against the rail selected as reference rail, usually the high rail of a curve. The wire chord stretched between A and D

\[
i = \frac{H_1}{H_2} = \frac{AB \times CD}{AB \times BD}
\]

\[
H_1 = \frac{AC \times CD}{2R}
\]

\[
H_2 = \frac{AB \times BD}{2R}
\]

**Fig. 2.12**

17 Para 2.2.2.2 (i) of IRTMM
represents the ‘Reference Line’ and the transmitting potentiometer (Transducer) which are fixed to the measuring trolley B and lining trolley C are connected to this wire by means of Forks and the wire drives.

The 4 Point lining method works on the principle that if we measure versine at two fixed places on circular curve on a fixed chord length, the ratio of versine i.e. H1 ÷ H2 will be constant. This relation of i = H1/H2 is true for all the circular curves irrespective of radius of curve. The machine measures versine at B i.e. H2 and calculates the theoretical versine at C (for circular curve H1 = H2 x i). If point A, B and D are on perfectly circular curve point C can be brought to the perfectly circular curve with the help of actual measurement of H2. However, since the points A and B only have been attended so far and the point C is being attended, therefore, point D is likely to be on disturbed track.

Presuming that the points A and B have been brought to perfectly circular curve alignment during tamping, but because of disturbed location of D, chord AD is shifted from ideal position. So the measurement of H2 as well as H1 is vitiated. If we tamp the curve as it is, we will not get a perfectly circular curve, but a curve (shown dotted) in between the existing curve and the design track (figure 2.13). The track so attended will have a residual error which will be equal to the distance by which point D is out of circular curve divided by n. Here n is the error reduction ratio and its value varies slightly from machine to machine. This is what we get when the smoothening mode of tamping is used.
2.8.2 Design mode in 4-point Lining System: If we want to get the designed track alignment as in design mode of tamping, the location of point D on the designed track alignment is required to be known. For this, the permanent Way Engineer carrying out the tamping must first measure the versines of the existing curve in advance and calculate the slews required at various stations on the curve to bring the same to design geometry. The slews calculated as above shall be written on sleeper top along with the direction of slew. During tamping, machine operator will feed the values written on track into the machine from the front cabin. The machine will electrically shift the point D the values fed and the machine will adjust value of H2 and accordingly H1. The error on account of the wrong position of point D will get rectified and we will get the design geometry at C. **This is what we get when design mode of tamping is used.**

4-point method of tamping is not used for straights. As soon as the front trolley approximately reaches tangent point, machine working shall be switched over to 4 point lining system and the same shall be continued beyond the end of curve upto another 20 m.

2.8.3 3-point lining system: The track is measured at three points B, C and D (Trolley A is folded) and lined according to specified theoretical versines. The ordinate at C is measured on chord BD and compared with preset ordinate value (for the circular curve). Any difference detected will activate lining control to effect the necessary slew. (Figure 2.14)

The 3-point method is mainly used if:- The track is to be lined according to specified radii or versines

2.8.3.1 Smoothening and design mode of 3 point lining: As discussed in 4 point lining trolley, D will normally be over disturbed track. If slew values are fed to bring trolley D to desired alignment position it becomes design mode else it is smoothening mode of tamping.

**Note for maintenance:**  *In open line, since the radius of track on curves is rarely uniform it is important to access the correct radius of curve and its transition length for getting the right version of curve. It is not advisable to go for 3– point lining in smoothening*

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18 Para 2.2.2.2 (ii) of IRTMM
mode. 3-point lining should always be done in design mode. The slew chart for achieving the final curve should be calculated. The version values should be fed as per final designed curve with slew values so calculated for achieving that curve. Best method of correcting curve is by 3-point design mode lining.

2.8.4 Tamping operations: For the curves, the datum rail for the cross level correction is the inner rail, which is given the general lift and the outer rail is lifted as per the desired superelevation. During tamping, however, the permanent way engineer has to associate with the machine operator and has to give three values regarding the curve to the machine operator who feeds them in the machine:

(a) Slews (b) Superelevation (c) Version correction for 4 point lining (d) Version value for 3-point lining

a) Slews: While working on curve in design mode, slews on the curve are to be entered in front cabin of the machine. The slews calculated for stations at 10m intervals shall be distributed for the sleepers in between assuming linear variation of slews between the stations. For the machines which tamp two sleepers at a time, the slew values shall be worked out and marked at every alternate sleeper. For the machines that tamp three sleepers at a time, the slew values shall be calculated and marked at every 3\textsuperscript{rd} sleeper. Arrow showing direction of slew shall also be marked on sleepers. 

Note: While working of the machine in smoothening mode, the slew values are not required to be worked out and marked on sleepers.
b) **Superelevation:** Throughout the transition curve, the super elevation keeps on changing. So the same should be written on the top of every alternate sleeper for the guidance of the machine operator. The super elevation in the circular curve remains constant which should also be written at regular spacing, say every 10<sup>th</sup> to 15<sup>th</sup> sleeper as reminder to the machine operator.

*Note: This is required whether the machine is working in design mode or smoothening mode.*

c) **Versine correction for 4 - point lining:** While working in 4 point mode as the front tower of the tamping machine enters the transition curve, the rear tightening bogie is still on straight. Therefore there is some error because of continuously changing curvature and certain adjustments have to be applied at the front bogie to compensate the error. Similar correction is required when the tamping machine leaves the curve at other end. These corrections are different for different machines and the appropriate charts are to be used by the machine operator for the same. The value of the transition correction depends on

- Degree of curve
- Length of transition
- Type of machine (Each machine has a separate chart for transition corrections)

The first two details are to be given by the permanent way engineer to the machine operator before the start of the work in the section so that the appropriate calculations can be made by the machine operator. The machine operator shall hand over the calculations to the permanent way engineer who shall get the same written on every alternate sleeper on the track. This transition curve versine correction is to be entered by machine operator in the machine. As per the principle of machine, the versine corrections are in outward direction on the first Transition curve (while entering from straight). On the other transition curve while entering from curve to transition (while exiting from curve) these versine corrections should be in inward direction.

*Note: The transition correction is required whether the machine is working in smoothening mode or in design mode. Providing the transition correction will eliminate the formation of goose neck type misalignment at the ends of transitions.*
(d) Versine value for 3-point lining: When working in 3-point lining, version value at every location is to be fed. The version value to be calculated for designed radius and transition length for a particular. Machine from the table given in each machine manufacturers manual.

2.9 Carrying out Realignment in Field: The realignment has to be carried out after the survey of the existing versines as per the para 2.2 above is done and the solution for the realignment is found by manual calculations/ computer programs described in chapter V. Thereafter, the following procedure shall be followed:

a) The reference pegs shall be erected on the cess at the stations where the curve is to be slewed. The reference pegs shall be rail/tie bar pieces or other appropriate material and suitably embedded in the cess so as not to get disturbed till the realignment work is done. The top of the reference peg shall be above the rail level. There shall be a paint mark/ saw cut at the top of reference peg from where the distances perpendicular to track can be measured.

b) The reference pegs shall be erected either at a constant distance from the existing center line of track or the actual distance from the existing center line shall be measured and recorded.

c) In double/multiple line section, the running rail of other track can also be used as reference.

d) The slew readings are to be written on the sleeper top along with the direction of slew. The readings at the stations are available from the calculations. Where track machines are to be used, the readings shall be linearly interpolated for the sleepers in between.

e) If slewing operation is done using track machines, the machine has to work in design mode and slews written on track are fed into the front tower of the machine by the machine operator. After tamping, the SE (PW) has to again measure the distance from the center line of the track to the reference peg. The difference in reading taken from existing center line of track and the reading after realignment will tell if the full value of slew has been achieved or not.

f) If slewing is done manually, the labour will use crow bars/jacks/TRALIS or similar equipment and be ready for the slewing operation. The measurement from the center line of track and the refer-
ence peg is taken during the slewing operation and the labour is
guided to stop when the desired value of slew has been achieved.

g) The reference pegs erected are helpful to know the retention of
the slews given. If required, the readings are taken from the
reference peg again after a few days to see whether the packing
has been done effectively and if the track is retaining the new
geometry or not.

h) Excessive Slews: If the slews in the track are excessive and
the same cannot be attained in one go by the track machine. In
such a case, the normal practice is to take up part of the slews
which are within practical limits of the machine packing, say 75-
100 mm maximum, and get the tamping done. The balance slews
shall be taken up in the second round. As we have studied in the
para 2.8.2 above, when we are working in the design mode, the
chord which is taken as reference for the alignment is electroni-
cally shifted so that the desired geometry is achieved. When we
give only part of the slews at some stations, the shifted chord is
not at the perfect circular geometry. In this situation, there is an
additional component of correction in alignment due to the normal
smoothening mode working of the machine (explained in para 2.8.1
above). Suppose the correction required at a station was 150mm
and the value aimed for correction is 50mm from the practical
consideration. In this case, when we measure the slew after the
first round of tamping, the actual slew by machine is more than 50
mm and residual slew required is less than the balance i.e. 100mm.
Since the residual slews are different at different stations, it will
be a good practice in open line that after the first round of
tamping, the survey of the versines shall be done again and a
new solution for the realignment problem shall be found out.
In the case of construction projects, where we are looking for per-
fekt circular curve, there is only one solution and the residual slew
values after a round of tamping may be used for the subsequent
round.

When working manually, the excessive slews may not be possible
in the open line as the trains have to continue to move on the
track. Here also, the curve may be taken up for correction in stages.

To slew the track, the ballast gives resistance at the sleeper ends.
For large slews in the track, therefore, it is desirable to open the
sleeper ends to reduce the resistance at the sleeper ends and also
to ensure that the track does not bounce back to its original geometry.

Immediately after heavy slewing, the track has a tendency to keep on returning to its original geometry. After such heavy slewing, it is desirable that one/two attentions are given at closer intervals to help retain the new alignment.

2.10 Long welded Rails (LWR) on curves: In para 2.3.1\textsuperscript{19} above, we have discussed the combined effect of curve and LWR. Due to this, the LWR is permitted in curves not sharper than 440 m radius in zone II, III & IV\textsuperscript{20} and up to 360 m radius in zone I\textsuperscript{21} with provision of extra ballast in shoulders. For reverse curves, maximum degree permitted is 2 degree (875m radius). RDSO has made a study vide Report No. CT-30 for conversion of LWR over curves sharper than 4\textdegree.

For LWR on curves, normal SEJ with straight tongue and stock rail shall not be laid on curves sharper than 0.5 degree as far as possible. For sharper curves, special drawings of SEJs have been issued by RDSO. The SEJ shall not be laid in transition portion of curves\textsuperscript{22}. Where laying of SEJ is not permitted or in sharper curves where special SEJ is not available, buffer rails may be provided as per provisions of LWR manual.

During realignment, the change in alignment in curves affects the stress free temperature of LWR. The shifting of track inwards (towards center of curve) will lead to the total track length reduced vis-à-vis the available rail length. Consequently, the stress free temperature gets lowered. On the other hand, shifting of track outwards will lead to the total track length getting increased vis-à-vis the available rail length and so, the stress free temperature gets increased. Therefore, after major realignment efforts, it will be a good maintenance practice to review the stress free temperature and carry out destressing, if required.\textsuperscript{22}

2.11 Rail Joints in Curves\textsuperscript{23}:

a) Rails joints on curves shall normally be laid square.

\textsuperscript{19} Para 3.2.1 of LWR Manual
\textsuperscript{20} Para 4.5.5 of LWR Manual
\textsuperscript{21} Para 6.3(iv) of LWR Manual
\textsuperscript{22} Para 424 and 425 of IRPWM
However, on the sharp curves less than 400 meters on the Broad Gauge and 300 meters on the Meter Gauge, elbows and kinks are likely to develop if rail joints are laid square. Therefore in this case, joints are laid staggerred.

b) On curved track, the length of outer rail is more than the length of inner rail. Therefore for same track length, the joints in inner rail gradually lead over the outer rail joints. When the inner rail of the curve is ahead of the outer rail by an amount equal to half the pitch of bolt holes, inner rail should be cut to obtain square joints. The excess length ‘d’ by which the inner rail gains over the outer rail is calculated by the formula –

\[
    d = \frac{LG}{R}
\]

where ‘d’ is the length in mm. by which the inner rail joint is ahead of the outer rail joint over the entire length of the curve, if cut rails are not provided.

\[ L = \text{length of the curve in meters} \]
\[ R = \text{radius of the curve in meters} \]
\[ G = \text{the gauge + width of the rail head in mm.} \]

The number of cut rails for a particular curve is worked out by the formula –

\[
    N = \frac{d}{\text{pitch of the bolt holes in mm}}
\]

It must be ensured that rail joints are square at beginning and at the end of the curve.

2.12 Indicators/Boards provided in curves\textsuperscript{24}:

2.12.1 Curve Board: Each approach of a curve should be provided with a curve board at the tangent point fixed on the outside of the curve. This Board should indicate the radius of the curve, the length of the curve, length of transition in metres and the maximum cant provided on the circular portion.

\textsuperscript{24} Para 409 (1) and 425 of IRPWM
of curve in millimetres in addition to the curve number and the track on which the curve is there. (Figure 2.15)

![Image of curve sign]

Fig. 2.15

2.12.2 Rail Posts Indicating Tangent Points: As per IRPWM, on the inside of the curve, rail posts should be erected on each approach of the curve, to indicate the positions of the beginning and end of transition curves. These rail posts may be painted in red and white colours respectively. In the case of non transitioned curve, similar rail post should be erected on the tangent track and on the circular curve over which the cant is run out, indicating the beginning and end of the virtual transition.

2.13 Extra clearances on Curves\textsuperscript{25}: On curves, additional clearances are to be given for the effect of the curvature, lean of the vehicle and swaying of the vehicle over and above sway already provided for in the clearance on straight tracks. On the inside of a curve, the extra clearance shall include the effect of curvature, the lean due to superelevation, and sway of the vehicles. The additional clearance to be given on the outside of a curve must allow for the effect of curvature only. Additional sway or lurch outwards due to the curve is within the inward lean of the vehicle due to superelevation, and the extra clearance due to lean is not required to be considered.

\textsuperscript{25} Appendix, Schedule II, BG SOD.
Note: The effect of the lean increasing the clearance outside the curve is not taken advantage of.

2.13.1 Allowance for curvature: The straight vehicle on the curve does not follow the curved path of the track but subtends a chord and the vehicle body is inwards or outwards of the track as shown in figure 2.17. The extra clearance is to be worked out for the longest vehicle plying on the section. As an example, the allowance for curvature for a vehicle 21340mm long, having 14785mm between bogie centre shall be calculated as under:
At the centre of vehicle (overthrow, $V_o$): The vehicle body projects inwards with a total amount equal to the versine on a chord equal to the center to center of the bogies. This extra clearance is to be provided on the inside of the curve. Using eq (1.2) for the versine in a curve,

$$V_o = \frac{14.785 \times 14.785 \times 1000}{8R} = \frac{27330}{R} \text{ mm} \quad \text{.........(2.3)}$$

Where $R$ is the radius of the curve in meters.

At the end of vehicle (end throw, $V_e$): The vehicle body projects outwards at ends by an amount equal to the difference in versine for the chord equal to the overall length of the vehicle minus the versine for the chord equal to the center to center of the bogies. This extra clearance is to be provided on outside of the curve. As above, using eq. (1.2),

$$V_e = \frac{21.340 \times 21.340 \times 1000}{8R} - \frac{14.785 \times 14.785 \times 1000}{8R}$$

$$V_e = \frac{29600}{R} \text{ mm} \quad \text{............................(2.4)}$$

2.13.2 Allowance for superelevation, lean (L): Due to superelevation, the vehicle leans inwards (Figure 2.18).
The extra clearance required (Lean) on inside of curve due to superelevation at any point at height ‘h’ above rail level is given by:

\[ L = \frac{h}{G} \times C_a \] (2.5)

where \( C_a \) is the actual superelevation, \( G \) is the dynamic gauge of the track.

In group A routes having potential for increasing speeds in future, during new works/ doubling etc, the maximum cant of 185 mm shall be assumed for locating all permanent structures and for transition lengths.\(^\text{26}\)

2.13.3 Allowance for additional sway on curves (S): The provision for additional lurch and sway on a curve is taken as one-fourth of the lean(L) due to superelevation.

2.13.4 Extra clearances on platforms: For platforms, the total additional clearance on account of curves to be provided is worked out as per above. It has been observed in that the

\(^{26}\) Note below Para 406(d)(1)(i) of IRPWM
clearances worked out as per the formulae are slightly on the higher side and on the platforms, the clearances beyond the limits can cause passengers to fall between track and the vehicle. As per Schedule of Dimensions, the clearances provided for the platforms, therefore, are slightly less than computed, and are worked out as follows:

(i) Inside of the Curve: \( V_o + L + S - 51 \)

\[ V_o + V_o + \frac{1}{4} L - 51 \] .........................(2.6)

(ii) On the outside of a curve: \( V_e - 25 \text{mm} \) .................(2.7)

2.13.5 Extra clearance between adjacent tracks: When trains are moving on parallel curved tracks, the worst case will be when the train on inner track is having end throw \( (V_e) \) towards the outer track and the train on outer track is having over throw \( (V_o) \) towards the inner track and both of these get added up. Further, the sway in both the vehicles can be in opposite direction, and therefore, double the normal sway is to be accounted for. However, nothing is allowed for lean due to superelevation, it being assumed that both tracks will be inclined the same amount.

The clearance between tracks is given by: \( V_e + V_o + 2 \times S \)

or \( V_e + V_o + 2 \left( \frac{L}{4} \right) \) .........................................................(2.8)

Note:

1) Based on above formulae, the extra clearances on account of the curves for the structures/ tracks have been worked out and given in Annexure II of the appendix of Schedule II in SOD.

2) Railway Board, vide letter No.68/WDO/SC/1 dated 16.04.1968, have issued instructions for increase of speed over curves for contemplating 160/200 kmph speed on Broad Gauge. As stated therein, while locating any permanent structures by the side of the track in the case of trunk routes and main lines which have the potential for the increase of speed in future, the need for additional clearances for realignment of curves for higher speed operation should be
kept in view. The particulars of the extra clearances necessary on curves between structures and the adjacent track and between tracks when there are no structures are given in additional appendix for extra clearances on curves for maximum speed of 200kmph. The same should be followed when high speeds of the order of 160-200kmph are contemplated.

2.14 Grade Compensation on Curves: Locomotive is the prime mover of a train. The locomotive exerts tractive effort (pulls) the train and the pull is transferred through coupling to the next vehicle and so on for the train as a whole to move. However, the vehicles on curve move in different direction compared to the locomotive. (Figure 2.19)

![Fig. 2.19](image)

There is small change in direction at end of each of the vehicle in curve. Correspondingly, there is change in direction of the pull on the vehicles moving on the curve. This change is achieved by the curving forces exerted by the rails on the vehicle. There is some loss of tractive effort (pull) exerted by the locomotive due to this change in direction. Therefore, the capacity of the locomotive to move a train gets reduced. This effect of the reduced hauling capacity of locomotive depends on the degree of curve (sharper curves have more loss of hauling capacity).
When a curve is present along with a gradient, to calculate whether the locomotive will be able to haul a train on the combination, the effect of the curve is expressed as equivalent gradient, which is to be added to the actual gradient. The capacity of the locomotive to haul a train depends on the gradient in the section, and accordingly the ruling gradients are specified for the sections or the locomotives are chosen depending on the ruling gradients and loads. When designing the curves, the effect of curve is to be compensated in (deducted from) the ruling gradient. This is called Grade Compensation. The grade compensation is specified as 0.04% per degree of curve for BG, which comes to 70/R percent.\textsuperscript{27} The same is specified as 0.03 % per degree of curve for MG (52.5/R percent) and 0.02% per degree of curve for NG 762 mm (35/R percent).

2.15 Visibility in Curves: Since there is a change in direction of track in curves, extra care is to be taken to ensure proper visibility on curves for the trains. This is also helpful during inspections by foot/push trolley. The visibility is especially important where signals, level crossings, work spots etc are located. On inside of curves, permanent structures shall be set back to ensure good visibility and the bushes/trees shall be periodically trimmed/cut to ensure visibility remains good.

2.16 Points and Crossings on Curves: The points and crossing shall be located on straight as far as possible. However, many a times these are required to be provided on curves due to space constraints. Points and crossings are a weak point in the track system due to the discontinuity in the running rail at the crossing, and transfer of the wheel from the running track to the tongue rail. In the points and crossings area, the 1 in 20 cant given in the rails is not provided and the behaviour of the vehicles on the points and crossings is different from that in the normal track. When the points and crossings are located in the curves, the problem of maintenance get compounded, and extra care has to be taken to ensure safety and the maintenance of the track parameters.

\textsuperscript{27} Para 418, IRPWM
The main line track of point is laid in curved alignment in such a case. When the train passes on the turnout side on a point and crossing, it experiences two curvatures, first one due to the curved alignment, and the second one due to the turnout. The vehicle travels on the dual curvature with effective curvature, which can be calculated as:

2.16.1 Similar Flexure: When turnout curve and main line curve are having same direction of curvature, it is called similar flexure (In Figure 2.20).

![Fig. 2.20](image)

**Turnouts in Similar Flexure and Contrary Flexure**

In this case, the curvatures of main line curve and turnout side are added up to find effective curvature for the train travelling on turnout side i.e.

\[
\frac{1}{Re} = \frac{1}{Rm} + \frac{1}{Rs}
\]

where 

- \(Re\) = Effective radius
- \(Rm\) = Main Line curve radius
- \(Rs\) = Switch or turnout side radius when laid in straight

\[
\therefore \frac{1}{Re} = \frac{Rm + Rs}{Rm \times Rs} \Rightarrow Re = \frac{Rm \times Rs}{Rm + Rs}
\]
2.16.2 Contrary flexure: When turnout curve and main line curve are having opposite directions of curvature, it is called contrary flexure (In Figure 2.20). In this case, the curvature of main line is deducted from the turnout side curvature to find out the effective curvature for the train travelling on turnout side.

\[
\frac{1}{R_e} = \frac{1}{R_s} - \frac{1}{R_m} \Rightarrow \frac{1}{R_e} = \frac{R_m - R_s}{R_s \times R_m}
\]

\[
\Rightarrow \frac{1}{R_e} = \frac{R_s \times R_m}{R_m - R_s}
\]

The curvature of the combined curve of radius \(R_e\) shall be in the direction of sharper curve (lower of \(R_m\) and \(R_s\) is sharper)

2.16.3 Superelevation for the Curves Having points and Crossings: In case of curves having points and crossings, the superelevation or cant provided shall be chosen keeping the trains moving on both the tracks in mind: on main line side as well as on turnout side. The cant to be provided shall ensure that the considerations of cant excess and cant deficiency are met for all the trains. It may be necessary to limit the amount of superelevation which can be provided considering the turnout side and this may limit the speed potential on the main line. Examples in Chapter III illustrate the procedure for computations for turnouts on curves.

2.16.4 Negative Cant: When the vehicle moves over a track where the outer rail is lower than the inner rail, the cant or super elevation is said to be negative. Such a situation occurs on points and crossings with contrary flexure. In such a case equilibrium cant, \(C\), is worked out for the speed on turnout side using equation (1.3), Chapter I, and the actual cant provided for mainline is 75-C on BG or 50-C on MG using the full cant deficiency permissible.\(^{29}\) The cant provided for main line becomes the negative cant for the turnout side as the curvature is of opposite direction in this case.

2.16.5 Laying Points and Crossing Sleepers in curve: The PSC sleeper layout for the points and crossing is also called fan shaped layout due to the changing spacing between sleepers on

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\(^{28}\) Para 411 of IRPWM

\(^{29}\) Para 413 of IRPWM
the LH rail and RH rails. This feature allows the same sleeper to be used for the LH turnout as well as RH turnout. The laying of the fan shaped layout in the curves requires the sleepers to be rotated as per the curvature in the main line and pre-curving the tongue and stock rails.

This requires change in sleeper spacing for which tables have been given by RDSO and may be referred to.

2.16.6 Change in Superelevation in Curves having Points and Crossings: There shall be no change in superelevation within 20 m outside the toe of switch and the nose of crossing on BG. For MG and NG, these values are 15 m and 12 m respectively. The points shall not normally take off from the transition portion of main line curve. However, in exceptional cases, if the same is unavoidable, special relaxation can be given by the Chief Engineer. The rate of change of cant and/or curvature in such a case shall be prescribed at the values allowed as per para 407 of IRPWM or lesser.

2.16.7 Additional Stipulations for Curves of similar flexure

a) Not followed by reverse curve - On a main line curve from which a curve of similar flexure takes off, not followed immediately by a reverse curve, the turn-out curve shall have the same cant as the main line curve.

b) Followed by reverse curve – A change of cant on the turn-out may be permitted starting behind the crossing and being run out at a rate not steeper than 2.8 mm per meter and subject to the maximum cant on the main line turn-out being limited to 65 mm on Broad Gauge

2.16.8 Loop line curvature: As discussed earlier, when points and crossing are laid in curve, the loop line experiences dual curvature, one due to the main line curve and the second due to the turnout itself. In such a situation, the turnout curvature gets added to the main track curvature. The points and crossings shall be laid such that the radius of lead portion on loop line is not less than

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30 Para 412 of IRPWM  
31 Para 414 of IRPWM  
32 Para 410(2) of IRPWM  
Para 415 of IRPWM
In exceptional cases, where the above radius of curvature is not possible, the minimum radius which can be permitted is 220 m on BG and 120 m on MG subject to:

- PSC/ST sleepers are provided in turn in curve to the same density as in main line.
- Full ballast profile as for track in main line

**2.16.9 Cross-Over on Curves:** In case of cross-over located in curves (Figure 2.21), the points and crossing in the inner curve takes off from the outside of the curve and the points and crossing in the outer curve takes off from the inside of the curve.

Using the equation (2.9), (2.10) and the various formulae given in the Chapter I, different values of the superelevation are obtained for the two points and crossing. The superelevation computed for the inner track having contrary flexure is less than that for the outer track having similar flexure. However, if we provide these different superelevations, twist will be there in the straight portion between the two crossings. Since the compound curvature of the crossover track is already very sharp and the length is
very small, this twist will be beyond permissible limits. Therefore, in case of cross over in curve, the complete cross over shall be in the same plane of inclination and the superelevation calculated for the inner track shall be provided for the complete cross over. Therefore, the speed restriction worked out for the inner track will be applicable for the outer track also. If the outer track cannot be raised in reference to the inner track, the cross over shall be laid without cant and the suitable speed restriction shall be imposed.

2.16.10 Curves with Diamond Crossings: Normally straight diamond crossings should not be provided in curves as these produce kinks in the curve and uniform curvature cannot be obtained. However, where provision of such diamonds cannot be avoided or in case where such diamonds already exists in the track, the approach curves of these diamonds should be laid without cant for a distance of at least 20 metres on either side of the diamond crossings. Cant should be uniformly runout at the rate specified in para 407 beyond 20 metres. The speed restrictions on the approach curve shall be decided in each case by the Chief Engineer taking into consideration the curvature, cant deficiency and lack of transition but shall in no case be more than 65 kmph in the case of Broad Gauge, 50 kmph in the case of Metre Gauge and 40 kmph in the case of Narrow Gauge (762 mm.) No speed restriction shall, however be imposed on the straight track on which the diamond is located. In the case of diamond crossings on a straight track located in the approach of a curve, a straight length of minimum 50 M between the curve and the heel of acute crossing of diamond is necessary for permitting unrestricted speed over the diamond, subject to maximum permissible speed over the curve from considerations of cant deficiency, transition length etc.

2.16.11 Maintaining Safety on Points and Crossing in Curve: In case of points and crossing with contrary flexure, there is discontinuity in the outer rail due to the presence of crossing. Since on curves, the wheel takes guidance from the outer rail, so it becomes very important to maintain the check rail opposite the crossing to maintain safety. Excess wear in the check rail, loosening of bolts of check rail and even

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33 Para 415 of IRPWM
34 Para 416 of IRPWM
breakage of the bolts is a common maintenance problem in such points and crossing. In case of the points and crossing with similar flexure, the wheel is always touching the outer rail where the tongue rail is also there. If extra efforts are not put in made to safeguard the tongue rail, it will very quickly get worn out/ chipped. The worn out/ chipped tongue rail may lead to improper guidance to the wheel and safety may be jeopardized. To safeguard the tongue rail in such situations, a 1 m to 1.5m long check rail is sometimes provided a little distance ahead of tongue rail even if the same is not required from curvature considerations.

2.17 Level Crossings in Curves: In case of level crossings on curves, the visibility for road users/trains is very important. Extra precautions are required to keep the visibility clear by regular cutting of bushes etc. The gate lodge shall preferably be located on the outside of the curve. If the gate lodge is to be located on inside of curve, it shall be so located as not to obstruct the visibility. Where the visibility is restricted due to curve for train/road user, warning bell operated by approaching train shall be provided and appropriate speed restriction for trains shall be imposed.

The gate leaves/lifting barriers shall be provided such that minimum distance from center of track is available at all points in curved track. The check rail to be provided on sharper curves shall require pre-curving. Since the track is superelevated, the road surface shall also have slope. In case of multiple lines, if the tracks are not in a single plane, there will be gradient in the road. The gradients in the road in such case shall be as gentle as possible.

2.18 Bridges in Curves: Normally we attempt to provide the bridges on straights and if required, the track on approaches is curved. However, there may be many difficult situations where the site conditions dictate that the bridges be provided on curved alignment. While providing curved alignment on bridges, the following are to be considered:

2.18.1 Alignment: If the length of bridge is small/degree of curve is small, the girders can be laid straight and the track on the bridge may be laid curved as shown in figure 2.22. In
this case, the girder shall be designed for the eccentricity due to the curvature.

![Diagram of girder and curve](image1)

**Fig. 2.22**

However, if the length of bridge is long or the curve is sharp, it is better to properly locate the girders so that the eccentricity of track on the girders is minimized. The girders shall be laid as chord in the curve, with the ends projecting out such as case is shown in figure 2.23. Total versine for the curved track in girder length L is given by equation (1.2) Chapter I, i.e. $L^2/8R$.

For minimum eccentricity of the track, girder ends shall be laid such that the theses lie approximately on the chords of the circular curve. The best condition will be to divide the eccentricity equally at the ends and center. If the same is not possible, the eccentricity will be equal to:

Eccentricity at center= $L^2/8R$ – Average of the eccentricity at the two ends

![Diagram of girder and curve](image2)

**Fig. 2.23**
2.18.2 **Laying of Steel channel sleepers on curves:** In the case of steel channel sleepers on steel girders laid in curves, the following factors have to be kept in mind:

a) **Eccentricity of Track on Girders:**
   - Any misalignment in the approach track shall be rectified. In case the curve requires realignment, the same may be done before the laying of steel channel sleepers is undertaken.
   - The existing bridge shall be surveyed and the exact value of eccentricity of the track on the girders shall be determined. If the girders are not in alignment as per the para 2.18.1 above, the rectification of the girder alignment may be undertaken before the laying of steel channel sleepers is commenced.
   - Precise values of the desired eccentricity of the track shall be worked out for each span at end and middle of girder. Based on the same, the actual eccentricity for each of the sleepers shall be worked out.
   - The steel channel sleepers as per standard RDSO drawing no B/1636/R2 are designed for no eccentricity of track. For eccentricity as in case of curved track, the sleeper will have to be checked/ redesigned. The maximum center to center distance between the rail bearers/ girder leaves supporting the channel sleepers as per above drawing is 1980 mm in BG. For the larger center to center distances, the channel sleepers have to be redesigned.

b) **Modifications in Track Fittings:**
   - There are two methods of laying the sleepers – Radial or Normal to track. Depending on the degree of curvature, the method of laying the sleepers has to be decided. For smaller degree of curve, the sleepers may be laid normal and slight angularity of the running rail may be accommodated in the track fittings, however for sharper curves, it will be better to go in for radial sleepers so that the fittings can function satisfactorily. In this case, the hook bolts will become inclined to the girder and this arrangement will have to be modified.
   - The fittings in the curved track are subjected to extra
loads in lateral direction due to the centrifugal / centripetal and curving forces. Therefore, the fittings have to be checked for these additional lateral loads.

c) Other issues in Sharper Curves:

- **Gauge:** The canted bearing plates and fittings have to be designed so as to achieve slack gauge, if required.

- **Check rails:** Arrangement/modification for providing check rails in curves may be required.

d) Fabrication of Steel Channel sleepers for Curves:

The sleepers are to be fabricated with differing eccentricity for each location. To achieve curved alignment, the difference in sleepers is in the location of hook bolt plate and the location of the stiffeners at the center of girder leaf/stringer.

To reduce the number of distinct sleepers being fabricated, the girders can be laid/corrected as per the figure 2.24. In this case, the dotted line shows the center line of the girder and the solid line shows the center line of the track. Sleepers on either side of the center line are mirror image of each other, and the sleepers in the middle quarter span are mirror image of the outer quarter span, albeit rotated by 180°. The sleepers in portion AD can be used in portion DC by just 180° rotation and putting in reverse order. Rotating by 180° and putting the sleepers in same order as in AD makes them fit for portion CE. Fixing the sleepers in reverse order to AD shall suffice for portion EB. This reduces the number of distinct sleepers to be fabricated for each span to 25%, and greatly helps in the laying of sleepers.

![Diagram](image-url)

**Fig. 2.24**
e) During the fabrication of the sleepers for multiple spans, the sleepers for the first span shall be brought to site with the hook bolt plates tack welded. After actual laying, the accuracy of the measurements and the calculations shall be checked. If there is no problem, the channel sleepers for the subsequent spans can be brought with fully welded hook bolt plates.

2.18.3 Superelevation in Track on Bridges in Curve: The superelevation is to be provided in the track to counteract the effect of change in alignment on the curve. The superelevation is to be provided either in the track or in the bearings of girders. The difference in the two methods is that if the super-elevation is provided in the girders at bearing level, the girders are not subjected to the full amount of lateral forces due to centrifugal forces. This aspect is to be decided during the design stage. If the super-elevation is to be provided in track, the following methods are used:

a) In ballasted deck girders: Only by increasing ballast cushion under the outer rail, superelevation can be provided in ballasted deck girder.

b) In unballasted deck girders:

- By appropriate packing beneath the sleeper, superelevation can be provided. The packing shall be trapezoidal in shape. If the bridge is in transition and the superelevation is to be provided in the steel channel sleepers, the packing plate will have to be different for each location depending on the superelevation. The sleepers and packing plates shall be properly marked for easy installation of the same at site.
- By appropriate shape of the wooden/ composite sleeper. In this case, each sleeper can be given appropriate shape to provide the superelevation. The transitions etc can be easily accommodated in the sleepers in this case.

2.19 Vertical Curves: The vertical curves are to be provided in the track where there is a change in gradients. Vertical curve is seen in figure 2.25 below.
The design considerations for the vertical curves have been discussed in para 1.12, Chapter I. The various maintenance issues related to the vertical curves are:

- **Drainage at junction of gradients**: On sag type of vertical curves, the drainage is a very important consideration and therefore, such curves shall not be located in cuttings or tunnels as far as possible.

- **Vertical curves in Long Tunnels**: In the long tunnels, the summit type vertical curves shall be avoided. The exhaust fumes, being hot and lighter than the surrounding air, will tend to collect near the crown of the tunnel and will create the problems of ventilation in long tunnels.

- **Vertical Curve in Horizontal Curve**: In horizontal curves, the vertical curves of summit type shall be avoided, especially where LWR is also there. In LWR, the track will tend to lift up on such summit curves. And, due to the horizontal curves, there are lateral forces as well as a tendency for off-loading of the vehicles. Therefore, the stability of the track under load will be adversely affected and track may be susceptible to buckling. Maintaining
the vertical curve in a horizontal curve will be much more difficult as track geometry changes in 3-dimensions.

- **Prohibited Locations for change in gradient:** Normally, there shall be no change in gradient in the points and crossing zone, on unballasted deck girder bridges, transition portion of horizontal curves etc and therefore, the vertical curves shall not be laid in such locations.

2.20 **Carrying out Works Requiring Speed Restriction for Long Duration on Curves:** In case some work is to be carried out in the vicinity of a curve, speed restriction may be required. If the duration of work is likely to be long, such as for yard remodelling or bridge rebuilding, the curve designed for the higher speeds will not be able to perform its functions properly. Since all the trains will be moving at the same restricted speed, the equilibrium speed will decrease. The vehicle will be un-necessarily moving on highly canted track for the duration of the work. To reduce the wear on inner rail and to improve safety/ the comfort on such curves, it is better to redesign the curve for the duration of the work and reduce the super-elevation of the curve in accordance with the requirement of the equilibrium speed after the speed restriction. The super-elevation can be restored to the original value after the work is over.

2.21 **Curves For high speed trains:** In case of cubic parabola, the versines at either end of the transitions are suddenly introduced/ removed. This causes a jerk at entry/exit of the transition. For high speed curves, this jerk becomes too high and the cubic parabola ceases to be suitable for such routes. Other shapes such as fourth order parabola, half sine curve and full sine curve are used as transitions in this case. On Indian Railways, for speeds up to 150kmph, the cubic parabola has been retained as the shape of transition without any problems. The curves for high speed trains are not discussed in this book as these have not yet seen the light of the day on Indian Railways.
2.22 CHAPTER II REVISION QUESTIONS

1. What is laid down inspection frequency of a curved track by SSE(P W), ADEN and SrDEN of section?

2. Why extra ballast shoulder shall be provided on outer side of curved track?

3. What are limits of gauge in initial laying and during maintenance on a curve of 300 M radius in BG?

4. How can we minimize wear on outer rail of curved track? What are the maximum limits of wear in rails as per IRPWM?

5. Discuss the benefits and problems associated with the check rails on sharp curves.

6. What extra clearances are required to be provided between:
   a. Parallel curved tracks.
   b. Curved track and platform.

7. What is grade compensation on curves? Why is this required?
CHAPTER III

CURVE DESIGN EXAMPLES
CHAPTER III

CURVE DESIGN EXAMPLES

3.1 Basics: Curves are to be designed as per the requirements of the passenger and goods traffic on the section. The theoretical aspect of various parameters of curve design have been discussed in Chapter I. The various principles laid down in the chapter I are used, and referred to wherever possible, while solving the examples. While designing the curves, the rounding off is done as follows:

a. The length of transition is rounded off to nearest 10 m, so as to decide the station numbers between which the transition is to be laid/maintained.

b. The superelevation calculated is rounded off to nearest 5 mm.¹

c. The speed potential for a curve is rounded off to the lower 5 KMPH.

3.2 Some observations on curve design: Table 3.1 below indicates calculated minimum values of radius in metres and the corresponding values of \( C_a \), \( C_d \) and \( C_{ex} \) for BG and MG with mixed traffic². For mixed traffic, these minimum values should be adhered to, if speed restrictions are to be avoided.

<table>
<thead>
<tr>
<th>( V_{max} ) KMPH</th>
<th>Minimum radius in metres</th>
<th>Gauge</th>
<th>( C_a ) (mm)</th>
<th>( C_d ) (mm)</th>
<th>( C_{ex} ) (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>640</td>
<td>B.G.</td>
<td>140</td>
<td>75</td>
<td>49.16</td>
</tr>
<tr>
<td>120</td>
<td>800</td>
<td>B.G.</td>
<td>147.68</td>
<td>100</td>
<td>75.00</td>
</tr>
<tr>
<td>130</td>
<td>997</td>
<td>B.G.</td>
<td>133.24</td>
<td>100</td>
<td>74.93</td>
</tr>
</tbody>
</table>

¹ Para 406 (1) (e) of IRPWM.
² The values of minimum radius for different speeds on BG have been worked out in example 3.1 below.
The above table is prepared to highlight the fact that even though the maximum cant permitted is upto 165 mm for BG, but the same does not help in permitting higher speeds on existing curves due to the cant excess limit being reached for the goods trains i.e. the full speed potential cannot be achieved in the mixed traffic conditions prevailing. To achieve the full potential, all the goods trains shall be speeded up or we shall construct a line for high speed passenger trains only. Adopting the limiting value of cant i.e. 165 mm, and the full value of cant deficiency of 75 mm (Not 100 mm, because for high speed trains, it is preferable to provide slightly lower cant deficiency from the passenger comfort point of view. The lateral acceleration is limited to 0.4 m/sec^2 in such cases). For 200 KMPH, with these parameters, we get the minimum radius of 2300 m (see example 3.2 below for calculations).

### 3.3 Curve design examples:

#### Example 3.1

Determine in each case the minimum radius on Board Gauge where maximum permissible speed on the section is:

- i) 100 KMPH
- ii) 120 KMPH
- iii) 130 KMPH
- iv) 160 KMPH

The booked speed of goods train may be assumed as 65 KMPH.

#### Solution:

Maximum permissible values of $C_a$, $C_d$ and $C_{ex}$ are as follows:\(^3\):

\(^3\) See paras 1.6.2.1, 1.6.2.2 and 1.6.2.3, Chapter I.
<table>
<thead>
<tr>
<th>Max. Speed KMPH</th>
<th>$C_a$ (mm)</th>
<th>$C_d$ (mm)</th>
<th>$C_{ex}$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>140</td>
<td>75</td>
<td>75</td>
</tr>
<tr>
<td>120</td>
<td>165</td>
<td>100</td>
<td>75</td>
</tr>
<tr>
<td>130</td>
<td>165</td>
<td>100</td>
<td>75</td>
</tr>
<tr>
<td>160</td>
<td>165</td>
<td>100</td>
<td>75</td>
</tr>
</tbody>
</table>

For a given set of values for maximum permissible $C_a$, $C_d$ and $C_{ex}$ and to permit unrestricted operation of passenger trains at the maximum permissible speed on the section, the acceptable minimum radius of a curve can be calculated as follows:

There are two unknowns, Radius and Cant to be provided. Assuming maximum permissible $C_d$ & $C_{ex}$ are reached, using the equation nos (1.6) and (1.8), Chapter I, we get

\[
\text{Cant to be provided} = \frac{13.76 V_m^2}{R} + C_{ex\ max}
\]

\[
\text{Cant to be provided} = \frac{13.76 V_m^2}{R} - C_{d\ max}
\]

\[
\therefore \frac{13.76 V_m^2}{R} + C_{ex\ max} = \frac{13.76 V_m^2}{R} - C_{d\ max}
\]

i.e. $R_{min1} = \frac{13.76(V_m^2 - V_G^2)}{C_{ex\ max} + C_{d\ max}}$ ..............................(3.1)

On consideration that maximum permissible limits of $C_a$ and $C_d$ are reached,

\[
C_{a\ max} = \frac{13.76 V_m^2}{R} - C_{d\ max}
\]

i.e. $R_{min2} = \frac{13.76 V_m^2}{[C_a] + [C_d]}$ ..............................(3.2)

The higher value of R given by eqn: (3.1) and (3.2) above is the limiting radius for operation of trains with unrestricted speed.
Case I: Speed of 100 KMPH:
Substituting the respective values from the table above, the equation (3.1) and (3.2) give:

\[ R_{\text{min}1} = 13.76 \frac{(100^2 - 65^2)}{75 + 75} = 529.76 \, m \]

\[ R_{\text{min}2} = \frac{13.76 \times 100^2}{140 + 75} = 640 \, m \]

Therefore the limiting radius \( R \) should be taken as 640 m. [Ans]

Case II: Speed of 120 KMPH:

\[ R_{\text{min}1} = \frac{13.76 \times (120^2 - 65^2)}{(75 + 100)} = 800.05 \, m \]

\[ R_{\text{min}2} = \frac{13.76 \times 120^2}{165 + 100} = 747.7 \, m \]

Therefore limiting radius \( R \) should be taken as 800.05 m, say 800 m. [Ans]

Case III: Speed of 130 KMPH:

\[ R_{\text{min}1} = \frac{13.76 \times (130^2 - 65^2)}{(75 + 100)} = 996.62 \, m \]

\[ R_{\text{min}2} = \frac{13.76 \times 130^2}{165 + 100} = 877.52 \, m \]

Therefore the limiting radius \( R \) should be taken as 996.62 m, say 1000m. [Ans]

Case IV: Speed of 160 KMPH:

\[ R_{\text{min}1} = \frac{13.76 \times (160^2 - 65^2)}{(75 + 100)} = 1680.7 \, m \]
\[ R_{\text{min}2} = \frac{13.76 \times 160^2}{(165 + 100)} = 1329.27 \text{ m} \]

Therefore the limiting radius \( R \) should be taken as 1680.7 m, say 1680m. [Ans]

Example 3.2

Find the minimum permissible radius on a Broad gauge high speed route (which is exclusively reserved for running of high speed passenger trains) to permit a maximum speed of 200 KMPH).

Solution:

Using equation (1.6), Chapter 1.

\[ C_a + C_d = \frac{G V_m^2}{g R} = \frac{13.76 \times V_m^2}{R} \]

Where \( V_m \) is in KMPH, \( R \) in metres, \( C_a \) and \( C_d \) are in mm.

Substituting values for \( C_a \), \( C_d \) and \( V_m \) as 165mm, 75mm \(^4\) & 200 KMPH respectively,

\[ (165 + 75) = 13.76 \times 200^2/R \]

Solving, \( R = 2293 \text{m}, \text{say 2300m} \)

So, a minimum radius of 2300m should be provided. [Ans]

Example 3.3

On Broad Gauge, a curve of 875 m radius is located on Rajdhani Route where the maximum permissible speed is 130 KMPH. Find out the cant, desirable and minimum length of transition for maximum permissible speed, assuming the speed of goods trains as 65 KMPH.

Solution:

For the above route limiting values prescribed are as under:

Cant = 165mm
Cant deficiency = 100mm

\(^4\) As discussed in para 3.2, the cant deficiency for higher speeds shall be limited to 75mm for better passenger comfort.
Desirable rate of change of cant/cant deficiency = 35mm/sec
Maximum rate of change of cant/cant deficiency = 55mm/sec

a) Cant deficiency consideration: Using equation (1.6),

\[ C_a + C_d = 13.76 \times \frac{V_m^2}{R} \]

\[ = 13.76 \times \frac{130 \times 130}{875} \]

\[ = 265.76 \text{ mm} \]

If we consider maximum value of \( C_d \) i.e. 100mm, actual cant works out to \((265.76 - 100) = 165.76 \text{ mm, say 165 mm, which is equal to the maximum value permitted.}\)

b) Cant excess consideration:

Using eq (1.3), Equilibrium cant for a speed of 65KMPH for goods trains

\[ = 13.76 \times \frac{V_G^2}{R} \]

\[ = \frac{13.76 \times 65 \times 65}{875} = 66.44 \text{ mm.} \]

If we add permissible cant excess i.e. 75mm, the maximum cant to be provided = 66.44 + 75 = 141.44 mm say 140 mm.

In order that values of cant deficiency and cant excess remain within permissible limits, minimum of the values computed above has to be adopted,

\text{i.e. adopt actual cant as 140 mm.}[Ans]

c) Maximum permissible speed

As per eqn (1.7), \( V_m = 0.27 \sqrt{(C_a+C_d)R} \)

\[ = 0.27 \sqrt{(140 + 100) \times 875} \]

\[ = 123.73 \text{ KMPH} \]
i.e. Maximum permissible speed is 120 KMPH (reducing to lower multiple of 5) [Ans]

(d) Desirable length of transition: As per eqn (1.12), (1.13) and (1.14), it shall be more than

\[ L_1 = 0.008 \times C_a \times V_m = 0.008 \times 140 \times 120 = 134.4 \text{ m}, \]
\[ L_2 = 0.008 \times C_d \times V_m = 0.008 \times 100 \times 120 = 96 \text{ m} \]
\[ L_3 = 0.72 \times C_a = 0.72 \times 140 = 100.8 \text{ m}. \]

Therefore, desirable length of transition shall be more than 134.4 m, rounded off to 140 m[Ans]

(e) Minimum length of transition: As per para 1.8.8, Chapter I, it shall be more than

\[ 2/3 \text{ of } L_1 = 2/3 \text{ of } 134.4 \text{ m} = 89.6 \text{ m} \]
\[ 2/3 \text{ of } L_2 = 2/3 \text{ of } 96 \text{ m} = 64 \text{ m} \]
\[ \frac{1}{2} \text{ of } L_3 = 1/2 \text{ of } 100.8 \text{ m} = 50.4 \text{ m}. \]

Therefore, minimum length of transition is more than 89.6 m, rounded off to 90 m[Ans]

Example 3.4

Determine the maximum permissible speed and desirable length of transition for a curve of 875 m radius on Broad Gauge where the cant is to be restricted to 75mm and the limiting value of \( C_d \) is also 75mm.

Solution:

As per equation (1.7), maximum permissible speed on the curve

\[ V_m = 0.27 \sqrt{(C_a + C_d) \times R} \]
\[ = 0.27 \sqrt{(75 + 75) \times 875} \]
\[ = 97.82, \text{ say 95KMPH (rounding off to the next lower multiple of 5) i.e. Maximum permissible speed on the curve is 95 KMPH [Ans]}. \]

(a) Desirable length of transition: As per eqn (1.12), (1.13) and (1.14), it shall be more than

\[ L_1 = 0.008 \times C_a \times V_m = 0.008 \times 75 \times 95 = 57 \text{ m} \]
\[ L_2 = 0.008 \times C_d \times V_m = 0.008 \times 75 \times 95 = 57 \text{ m} \text{ and} \]
\[ L_3 = 0.72C_a = 0.72 \times 75 = 54 \text{ m}. \]

\[ \Rightarrow \text{ Desirable length of transition shall be more than 57 m, rounded off to 60m [Ans]} \]

**Example 3.5**

Find out the maximum permissible speed on a curve of 600m radius on a Broad Gauge New Delhi – Mumbai Central Rajdhani route, when the length of available transition is restricted to 50 metres.

**Solution:**

For the above route, limiting values prescribed are as under

- Cant = 165mm
- Cant deficiency = 100mm
- Cant excess = 75mm
- Rate of change of cant / cant deficiency in exceptional circumstances = 55mm/sec
- Steepest cant gradient = 1 in 360

a) Cant deficiency consideration: Using Eq (1.6),

\[
C_a + C_d = 13.76 \times \frac{V_m^2}{R}
\]

\[
= 13.76 \times \frac{120 \times 120}{600}
\]

\[= 330.24 \text{ mm} \]

With a maximum value of \(C_d = 100\text{mm}\) the actual cant works out to 330.24 – 100 = 230.24 which is > 165mm and hence cant is to be limited to 165mm.

b) Cant excess consideration: Using Eq (1.3),

Equilibrium cant for a speed of 65KMPH for goods trains

\[
= 13.76 \times \frac{V_G^2}{R}
\]

\[= 96.89 \text{ mm}. \]

With 165mm cant, cant excess works out to

\[= 165-96.89 = 68.11 \text{ mm} \]
= 68.11 mm < 75mm. Hence 165 mm cant found out in a) above is permissible

c) With a cant of 165mm and cant deficiency of 100mm maximum permissible speed as per eq (1.7):
\[ V_m = 0.27 \sqrt{(C_a + C_d)R} \]
\[ = 0.27 \sqrt{(165 + 100)600} \]
\[ = 107.66\text{KMPH say 105KMPH (rounding off to next lower multiple of 5)} \]

As per para 1.8.8, Chapter I, minimum length of transition would be more than the following three values:

a) \[ \frac{2}{3} \text{ of } L_1 = \frac{2}{3} \times 0.008 \times C_a \times V_m \]
\[ = \frac{2}{3} \times 0.008 \times 165 \times 105 = 92.4\text{m or} \]

b) \[ \frac{2}{3} \text{ of } L_2 = \frac{2}{3} \times 0.008 \times C_d \times V_m \]
\[ = \frac{2}{3} \times 0.008 \times 100 \times 105 = 56\text{ m or} \]

c) \[ \frac{1}{2} \text{ of } L_3 = \frac{1}{2} \times 0.72 \times C_a \]
\[ = \frac{1}{2} \times 0.72 \times 165 = 59.4\text{ m.} \]
\[ \therefore \text{Minimum length of transition} = 92.4\text{m} \]

Since available length of transition is limited to 50m which is less than the minimum length of 92.4m, the cant and cant deficiency values have to be restricted⁵. For finding the optimum values of cant and cant deficiency, the expression for speed on the transition curve should be equated to the expression for speed on the circular curve.

Further, for optimizing the length of the transition, cant and cant deficiency should be equal, i.e. \( C_a = C_d \).

Transposing equation (1.13),
\[ V_m = \frac{r_{ca} \times L}{C_a} \]

Here \( V_m \) is in m/sec. To convert the same into KMPH, a factor of 3.6 is to be used, i.e.
\[ V_m = \frac{r_{ca} \times L}{C_a} \times 3.6 \] \[ \text{…………………..(3.3)} \]

⁵ See para 1.8.9, Chapter I.
Also, from equation (1.7), \( V_m = 0.27\sqrt{C_a + C_d} \)  
……..……(3.4)  

Since the length of transition is restricted, the limiting value of \( r_{ca} \) i.e. 55 mm/sec shall be used. Substituting \( C_a = C_d \), and equating the two values of \( V_m \) from equations (3.3) and (3.4), we get  
\[
\frac{55 \times 50}{C_a} \times 3.6 = 0.27\sqrt{2C_a \times 600}
\]

\[
C_a = \left[ \frac{(55)^2 \times (50)^2 \times (3.6)^2}{(0.27)^2 \times 2 \times 600} \right]^{1/3}
\]
i.e. \( C_a = 103.86 \) mm, rounded off to 105 mm.  

Since the value 103.86 mm is more than the limiting value of \( C_d \), the same is taken as 100 mm.  

Cant gradient for 100 mm cant and length of transition of 50m is 100 mm/ 50 m, i.e. 1 in 500. This is flatter than maximum permissible 1 in 360, hence O.K.  

Using eq (1.7), maximum speed on curve  
\[
V_m = 0.27 \sqrt{(105 + 100) \times 600}
\]

= 94.69 KMPH rounded off to 90 KMPH. (can also be rounded off to 95 KMPH)  

**Therefore, maximum permissible speed on the curve when the transition length is restricted to 50 m is 90 KMPH.  [Ans].**

**Example 3.6**  
Determine the maximum permissible speed for a non transitioned curve of 1000 m radius on Broad Gauge when the maximum sectional speed is 100 KMPH.  

**Solution:**  
On a non-transitioned curve, the concept of virtual transition will hold good\(^6\).  

Now, Length of virtual transition for B.G. = 14.6m.  

As in the above example, equating the speed calculated in equation (3.3) with that in equation (3.4) above and assuming \( C_a = C_d \), \( C_a \) and \( C_d \) shall be within limits.)

---

\(^6\) Para 1.11.1, Chapter I.
\[
\frac{14.6 \times 55 \times 3.6}{C_a} = 0.27 \sqrt{(C_a + C_d) \times 1000}
\]

Squaring both the sides,

\[
\frac{(14.6)^2 \times (55)^2 \times (3.6)^2}{C_a^2} = (0.27)^2 \times C_a \times 2 \times 1000
\]

\[
C_a = \left[ \frac{(14.6)^2 \times (55)^2 \times (3.6)^2}{(0.27)^2 \times 2 \times 1000} \right]^{1/3}
\]

\[
C_a = 38.56 \text{mm, rounded off to 40mm.}
\]

Take \(C_d\) as 38.56 mm as calculated.

Therefore, Maximum permissible speed

\[
V_m = 0.27 \sqrt{(C_a + C_d) R}
\]

\[
= 0.27 \times \sqrt{(40 + 38.56) \times 1000}
\]

\[
= 75.68 \text{ KMPH}
\]

say 75 KMPH (rounding to lower multiple of 5)

Cant gradient, \(i = 1 \text{ in } \frac{40}{14600} \)

\[
= 1 \text{ in 365, flatter than 1 in 360, hence permissible.}
\]

**Therefore, the speed on non transitioned curve will be 75 KMPH. [Ans]**

However as mentioned in para 1.11, chapter I, it is always desirable to actually provide some transition. For instance, in the present example, an actual transition length of 14.6m can easily be provided, as the shift of the circular curve using equation (1.21) will be:

\[
\text{Shift} = \frac{L^2}{24R} = \frac{14.6^2 \times 1000}{24 \times 1000} \text{ mm} = 8.9 \text{ mm say 9 mm. This much shift can be easily accommodated even in most difficult layouts.}
\]
Example 3.7
In the example 3.6, determine the maximum permissible speed if the curve is without cant.

Solution:
For the above route the limiting value of cant deficiency is 75mm and the maximum rate of change of cant/cant deficiency will be 55mm/sec.

Since the curve is without cant, \( C_a = 0 \)

Proceeding in the manner of the above example, we get

\[
\frac{L \times 55 \times 3.6}{C_d} = 0.27 \sqrt{(C_a + C_d) \times R}
\]

Squaring both sides and putting in known values,

\[
\frac{(14.6)^2 \times (55)^2 \times (3.6)^2}{C_d^2} = 0.27^2 \times (C_d) \times 1000
\]

\[
C_d = \left[ \frac{(14.6)^2 \times (55)^2 \times (3.6)^2}{(0.27)^2 \times 1000} \right]^{1/3}
\]

i.e. \( C_d = 48.58 \)mm

Maximum permissible speed, \( V_m = 0.27 \sqrt{(C_a + C_d)R} \)

i.e. \( V_m = 0.27 \sqrt{(0 + 48.58) \times 1000} \)

i.e. \( V_m = 59.51 \) KMPH, rounded off to 55 KMPH \{Rounding off in this can also be done to 60 KMPH\}

Rate of change of cant deficiency will be:

\[
r_{cd} = \frac{C_d}{L/V} = \frac{48.58}{14.6} \left[ \frac{55}{3.6} \right]
\]

= 50.83mm/sec, which is less than the maximum permissible, i.e. 55 mm/sec, hence OK.

Therefore, the speed for non transitioned curve without superelevation is 55 KMPH.\{Ans\}
Example 3.8
For a compound curve of radii 700 m and 1000 m on BG find out the equilibrium cant to be provided and determine the minimum length of transition to be provided between the two circular curves when the sectional speed is 100 KMPH and the booked speed of the goods train is 65 KMPH.

Solution:
Let the curve of 700 m radius be designated as curve no. 1, and the 1000 m radius curve be designated as curve no 2. And the actual cants for the two curves be designated as \( C_{a1} \) and \( C_{a2} \) whereas the cant deficiencies for the two curves be designated as \( C_{d1} \) and \( C_{d2} \).

a) Cant deficiency consideration: For curve no 1 of 700m radius for 100KMPH, the cant for maximum speed (which is equal to \( C_a + C_d \)) required is

\[
C_{a1} + C_{d1} = 13.76 \frac{V_m^2}{R_l}
\]

\[
= 13.76 \times \frac{100 \times 100}{700}
\]

\[
= 196.6 \text{ mm} \quad (3.5)
\]

For curve no 2 of 1000m radius, for 100 KMPH the cant, for maximum speed required is

\[
C_{a2} + C_{d2} = 13.76 \frac{V_m^2}{R_2}
\]

\[
= 13.76 \times \frac{100 \times 100}{1000}
\]

\[
= 137.6 \text{ mm} \quad (3.6)
\]

b) On a compound curve, given certain values of \( (C_{a1} + C_{d1}) \) and \( (C_{a2} + C_{d2}) \) from equations (3.5) and (3.6), the optimum values of \( C_{a1}, C_{d1}, C_{a2}, \) and \( C_{d2} \) shall be such as to give the minimum lengths of transitions between the compound curves.

For optimizing the length of transition between the compound circular curves, \( C_{a1} - C_{a2} \), should be equal to \( C_{d1} - C_{d2} \).
i.e. \( C_{a1} - C_{a2} = C_{d1} - C_{d2} \) ..............................................(3.7)

Now, \( C_{a1} - C_{a2} = \frac{(C_{a1} - C_{a2}) + (C_{d1} - C_{d2})}{2} \)

Using equation (3.7), \( C_{a1} - C_{a2} = \frac{(C_{a1} - C_{a2}) + (C_{d1} - C_{d2})}{2} \)

\[ = \frac{(C_{a1} + C_{d1}) - (C_{a2} + C_{d2})}{2} \]

Using values in equation (3.5) and (3.6), \( C_{a1} - C_{a2} = \frac{196.6 - 137.6}{2} = 29.5 \text{ mm, say 30 mm.} \)

i.e. \( C_{a1} - C_{a2} = C_{d1} - C_{d2} = 30 \text{ mm} \).................................(3.8)

**c)** The length of transition at the either end of the compound curve will be minimum when the difference \( C_{a1} \) and \( C_{d1} \) / \( C_{a2} \) and \( C_{d2} \) is the least possible.

To ensure this, adopt cant deficiency for sharper curve, \( C_{d1} = 75 \text{ mm} \) i.e. the maximum permissible. (Since \( C_{d1} \) is higher than \( C_{d2} \))

Then using equation (3.5), \( C_{a1} = 196.6 - 75 = 121.6 \text{ mm, rounded} \)

off as 120 mm.................................................................(3.9)

Using equation (3.8), \( C_{a2} = 120 - 30 = 90 \text{ mm} \).................................(3.10)

\( C_{d2} = 75 - 30 = 45 \text{ mm} \)..............................................(3.11)

**d) Cant excess consideration:** For curve no 1, 700 m radius equilibrium cant for 65KMPH

\[ = 13.76 \frac{V_G^2}{R} \]

\[ = 13.76 \times \frac{65 \times 65}{700} \]

\[ = 83.05 \text{ mm.} \]

From equation (3.9), \( C_{a1} \) is 120 mm, the cant excess works out to 120 mm – 83.05 mm = 36.95 mm, which is less than the maximum permissible i.e. 75 mm, hence OK.

For curve no 2, 1000m radius, equilibrium cant for 65KMPH
\[ V_G^2 = \frac{13.76 \times 65 \times 65}{1000} \]

\[ = 58.14 \text{ mm.} \]

From equation (3.10), \( C_{a_2} \) is 90 mm, the cant excess works out to 90 mm – 58.14 mm = 31.86 mm, which is less than the maximum permissible i.e. 75 mm, hence OK.

e) Using formulae (1.15), (1.16) and (1.17), and values given in (3.9), (3.10) and (3.11), the length of transition required between the compound circular curves shall be the more than the following:

\[ L_1 = 0.008 \times (C_{a_1} - C_{a_2}) \times V_m \]

\[ = 0.008 \times (120 - 90) \times 100 \]

\[ = 24 \text{ m} \]

\[ L_2 = 0.008 \times (C_{d_1} - C_{d_2}) \times V_m \]

\[ = 0.008 \times (75 - 45) \times 100 \]

\[ = 24 \text{ m} \]

\[ L_3 = 0.72 \times (C_{a_1} - C_{a_2}) \]

\[ = 0.72 \times (120 - 90) \]

\[ = 21.6 \text{ m} \]

Therefore, adopt the transition length between the compound curve as 30 m. [Ans]

Example 3.9

Find out the superelevation and transition length to be provided on a Board Gauge curve of 500m radius located in a hilly section where the pattern of traffic is as follows:

<table>
<thead>
<tr>
<th>Sr. No.</th>
<th>Type of trains</th>
<th>No. (n)</th>
<th>Load(Tonne) (w)</th>
<th>Speed (KMPH) (V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Mail/Express and Passenger Trains</td>
<td>12</td>
<td>750</td>
<td>50</td>
</tr>
<tr>
<td>2</td>
<td>Goods Traffic</td>
<td>7</td>
<td>1700</td>
<td>15</td>
</tr>
<tr>
<td>3</td>
<td>Mixed Trains</td>
<td>2</td>
<td>500</td>
<td>30</td>
</tr>
</tbody>
</table>
Solution:

This section carries mixed traffic and choosing proper cant corresponding to equilibrium speed shall help reduce maintenance efforts\(^7\). The cant chosen should be such that it neither affects the operation of fastest trains on the section due to high cant deficiency nor involves a very high cant excess for the slow moving trains, leading to extra maintenance problems due to heavy wear on the inner rails of curves.

a) To ensure the above, the equilibrium speed is determined using Russian formula (1.4):

\[
V_{eq} = \sqrt{\frac{n_1 w_1 V_1^2 + n_2 w_2 V_2^2 + n_3 w_3 V_3^2}{(n_1 w_1 + n_2 w_2 + n_3 w_3)}}
\]

where:

- \(n_1\), \(n_2\), \(n_3\), denote the number of trains in each speed group.
- \(w_1\), \(w_2\), \(w_3\) denote the total weight of each train failing in each speed group.

For the present case,

<table>
<thead>
<tr>
<th>(n)</th>
<th>(w)</th>
<th>(v)</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>750t</td>
<td>50KMPH</td>
</tr>
<tr>
<td>7</td>
<td>1700t</td>
<td>15KMPH</td>
</tr>
<tr>
<td>2</td>
<td>500t</td>
<td>30KMPH</td>
</tr>
</tbody>
</table>

Substituting the above values in the formula:

\[
V_{eq} = \sqrt{\frac{(12 \times 750 \times 50^2) + (7 \times 1700 \times 15^2) + (2 \times 500 \times 30^2)}{(12 \times 750) + (7 \times 1700) + (2 \times 500)}}
\]

\(= 34.5\) KMPH.

b) **Equilibrium Cant**: Using equation (1.3), for this equilibrium speed of 34.5 KMPH, cant required on a curve of 500m radius is:

\(13.76 \times 34.5^2/500 = 32.76\) mm, so adopt a cant of 35mm

c) **Cant Deficiency Considerations**: Cant required for the fastest trains (at 50 KMPH)

\(^7\) Para 1.4.7, Chapter 1.
\[ = 13.76 \times 50^2/500 = 68.8 \text{ mm} \]
\[ \therefore \text{Maximum cant deficiency} = 68.8 - 35 = 33.8 \text{ mm} < 75\text{mm}, \text{ hence OK} \]

d) **Cant Excess Considerations:** Cant required for the loaded goods trains (at 15 KMPH)
\[ = 13.76 \times 15^2/500 = 6.19 \text{ mm} \]
\[ \therefore \text{Maximum cant excess} = 35 - 6.19 = 28.81\text{mm} < 75\text{mm}, \text{ hence OK} \]

**Hence the superelevation to be provided: 35 mm. [Ans]**

e) **Transition length**
Using equations (1.12), (1.13) and (1.14) desirable transition length required shall be more than the following three values:
\[ L_1 = 0.008 \times C_a \times V_m = 0.008 \times 35 \times 50 = 14\text{m} \]
\[ L_2 = 0.008 \times C_d \times V_m = 0.008 \times 33.8 \times 50 = 13.52\text{m} \]
\[ L_3 = 0.72 \times C_a = 0.72 \times 35 = 25.2\text{m} \]

**Provide a transition length of 30m. [Ans]**

**Example 3.10**
A 1 in 8½ turnout takes off from the outside of Broad Gauge curve of 600m radius on Rajdhani route, with a maximum speed of 130 KMPH. Determine the cant to be provided and also the permissible speed for Rajdhani Express and for the other trains, on the main line curve. The speed of 30 KMPH shall be provided on turnout.

**Solution:**
a) **Equivalent Radius:** To find the effective radius for train travelling on loop line side of the 1 in 8½ turnout taking off on the outside of a 600m radius curve, we use equation (2.10),
\[ R_e = \frac{R_m \times R_s}{R_m - R_s} \]

Where
\[ R_m = \text{Radius of main line curve i.e. 600m} \]
\[ R_s = \text{Radius of a 1 in 8½ turnout curve taking off from the straight i.e. 232.26m} \]
With the above formula

\[ R_e = \frac{600 \times 232.26}{600 - 232.26} \]

= 378.95 m

b) **Cant to be provided:** Cant required for turnouts per item 22 of Chapter 11 of Schedule of Dimensions (B.G.)

\[ C = \frac{GV^2}{127 \times R} \]

\[ = \frac{1750 \times 30^2}{127 \times 378.95} \]

= 32.73 mm

Considering the full amount of cant deficiency of 75 mm, positive cant to be provided on main line = 75 – 32.73 = 42.27 mm, say 40 mm.

c) **Speed for Main Line:** Permissible speed on main line for high speed Rajdhani Express \( (C_d = 100 \text{mm}) \)

\[ V_m = 0.27 \times \sqrt{(C_a + C_d)} R \]

= 0.27 \times \sqrt{(40 + 100) \times 600}

= 78.25 KMPH say 75 KMPH[Ans]

Permissible speed for trains other than Rajdhani Express \( (C_d = 75 \text{mm}) \)

\[ V_m = 0.27 \times \sqrt{(C_a + C_d) \times R} \]

= 0.27 \times \sqrt{(40 + 75) \times 600}

= 70.92 KMPH or say 70 KMPH[Ans]

*Note: It may be seen that if we reduce the speed on the turnout side, the allowable superelevation on the main line side increases, and the speed on main line also increases.*

---

8 Formula after correction slip no. 4 to Schedule of Dimensions.
Example 3.11
A 1 in 12 turnout takes off from the inside of a 800m radius Broad Gauge curve where the maximum sectional speed is 100KMPH. Determine the cant and maximum permissible speed when
A) The turnout is not followed by a reverse curve
B) The turnout is followed by a reverse curve
Allowable speed on the turnout side is 15 KMPH.
Solution:
Case(A) Turnout not followed by a reverse curve:
a) Equivalent radius: Using equation (2.9), the effective radius of 1 in 12 turnout curve on the inside of a 800m radius curve:
\[ R_e = \frac{R_m \times R_s}{R_m + R_s} \]
\[ R_m = \text{Radius of main line curve, i.e. 800 m.} \]
\[ R_s = \text{Radius of switch, 441.36 m} \]
\[ R_e = \frac{800 \times 441.36}{800 + 441.36} = 284.44 \text{ m.} \]
b) Cant for turnout: Equilibrium cant for turnout curve at speed of 15 KMPH
\[ = 13.76 \times \frac{V^2}{R_e} \]
\[ = 13.76 \times \frac{15 \times 15}{284.44} \]
\[ = 10.88 \text{ mm, say 10mm} \]
c) Cant for mainline: With cant excess of 75 mm, the actual cant permissible will be 10 mm + 75 mm i.e. 85mm.[Ans]
d) Maximum Permissible Speed for Main Line: Using equation (1.7), and considered the maximum cant deficiency of 75 mm, maximum permissible speed on main line with a cant of 85mm is:
\[ V_m = 0.27^* \sqrt{(C_a + C_d) \times R} \]
\[ = 0.27^* \sqrt{(85 + 75) \times 800} \]
\[ = 96.59 \text{ KMPH, rounded off to 95 KMPH.}[\text{Ans}] \]

**Case (B) Turnout followed reverse curve:**

In this case, the maximum permissible cant on main line is 65mm\(^9\). The calculated value is more than the maximum allowed value, so **65 mm cant is adopted.**[Ans]

Maximum permissible speed on main line:

\[ V_m = 0.27^* \sqrt{(C_a + C_d) \times R} \]
\[ = 0.27^* \sqrt{(65 + 75) \times 800} \]
\[ = 90.36, \text{ rounded off to 90 KMPH.}[\text{Ans}] \]

**Example 3.12**

A 1 in 12 turnout takes off from the outside of a 350m radius Broad Gauge curve. Find out the maximum permissible speed over the main line, if the permissible speed over the turnout is 30 KMPH.

**Solution:**

The lead radius of a 1 in 12 turnout with curved switch taking off from straight is 441.36m. The equivalent curvature for the turnout is to be determined from the formula (2.10) which is for the contrary flexure case. Therefore, the effective radius of the 1 in 12 turnout with curved switch taking off on the outside of a 350m radius curve is

\[ R_e = \frac{R_m \times R_s}{R_m - R_s} \]

\( R_m = \text{Radius of the main line curve} = 350m \)

\( R_s = \text{Radius of the 1 in 12 turnout with curved switch taking off from straight} = 441.36 m \)

With the above formula,

Equilibrium radius \( R_e = 350 \times 441.36 / (441.36 - 350) = 1690.85m \)

However as discussed in para 2.16.2, chapter II, since lead

\(^9\) Para 2.16.7(b) of Chapter II.
curve \( R_s = 441.36 \text{m} \) is flatter than the main line curve \( R_m = 350 \text{m} \), the equivalent turnout curve will have curvature in the same direction as the main line track, even though the curve is taking off from the outside of the curve. This is also clear from the figure 3.1 below.

![1 in 12 Turnout with curved switch](image)

**Fig. 3.1**

Equilibrium cant for this speed is:

\[
V = 13.76 \times \frac{V^2}{R_c}
\]

\[
= 13.76 \times \frac{30 \times 30}{1690.85}
\]

\[
= 7.32 \text{ mm}
\]

The cant computed above is not negative in the case and cant excess is to be considered. With cant excess of 75 mm, the actual cant to be provided for turnout will be \( 7.32 + 75 = 82.32 \text{ mm} \), say 80 mm.

Maximum permissible speed on the main line curve with above cant and a cant deficiency of 75 mm will be

\[
V_m = 0.27 \sqrt{(C_a + C_d) \times R}
\]

\[
= 0.27 \sqrt{(80 + 75) \times 350}
\]

= 62.88 KMPH, say 60 KMPH.

Hence the main line speed has to be restricted to 60 KMPH. [Ans]
Example 3.13
A 1 in 12 crossover is laid between two curved parallel tracks having a radius of 600 m. Determine the cant to be provided and maximum permissible speed on the two main line tracks. The sectional speed is 130 KMPH. The speed over the cross over is 30 KMPH.

Solution:

Fig. 3.2

i) **Location A**

a) **Effective Radius:** The effective radius of the 1 in 12 turnout curve at Location A where there is contrary flexure turnout is given by formula (2.10):

\[ R_e = \frac{R_m \times R_s}{R_m - R_s} \]

Where

- \( R_s = \) Effective radius of turnout curve, 441.36 m
- \( R_m = \) Radius of main line curve, 600 m.

\[ R_e = \frac{600 \times 441.36}{600 - 441.36} = 1669.29 \text{ m say 1670 m} \]

b) **Cant to be Provided on Main Line:** Cant required for a curve of 1670 m radius as per item 22 of Chapter II of Schedule of Dimension (B.G.):

\[ = 13.76 \times \frac{V^2}{R_e} \]

\[ = 13.76 \times \frac{30 \times 30}{1670} = 7.4 \text{ mm} \]
Cant to be provided on main line, with full value of cant deficiency of 75 mm for the turnout = 75 – 7.4 = 67.6 mm, say 70 mm

c) **Permissible Speed:**

Permissible speed for high speed trains:

\[ V_m = 0.27 \sqrt{(C_a + C_d) \times R} \]

\[ = 0.27 \sqrt{(65 + 100) \times 600} \]

= 86.23 KMPH, say **85 KMPH**

Permissible speed for other than high speed trains:

\[ V_m = 0.27 \sqrt{(C_a + C_d) \times R} \]

\[ = 0.27 \sqrt{(65 + 75) \times 600} \]

= 79.63 KMPH say **80 KMPH**  [Ans.]

ii) **Location B**

The turnout is having similar flexure here. The speed potential for the similar flexure turnout is more than that for the contrary flexure turnout, as higher cant can be provided. However, as discussed in para 2.16.9, Chapter II, the superelevation has to be kept the same for the location A as well as location B. **The same cant and speed as worked out for location A should be adopted.**  [Ans.]

The entire cross over shall be laid in the same inclined plane and the outer track shall be raised with respect to the inner one for the same. Where this is not possible, both the tracks should be laid in the same plane with whatever cant is permissible. Otherwise the turnouts shall be provided with lesser cant (may be even zero cant) and suitable speed restriction shall be imposed.

**Example 3.14**

An existing 600 m radius curve on existing Meter Gauge route is provided with a cant of 50 mm and a transition of 40 m on either side. The curve has to be redesigned for sectional speed of passenger trains at 100 KMPH. The booked speed of goods train is 50 KMPH. Find out
i) The cant and maximum permissible speed on the above curve for passenger trains.

ii) Transition length required.

iii) The cant to be provided and the maximum permissible speed if the length of transition is kept at 40m only.

**Solution:**

i) Check for $V_m = 100$ KMPH.

For MG, the maximum cant to be provided: 100 mm, maximum cant deficiency permissible: 65 mm, Maximum cant excess permissible: 50 mm.

a) Cant for 100 KMPH,

$$C_a + C_d = \frac{1058}{9.81} \times \frac{(V_m/3.6)^2}{R}$$

$$= 8.32* \frac{100*100}{600}$$

$$= 138.67 \text{mm}$$

With maximum permissible $C_d = 50 \text{mm}$, $C_a$ works out to $138.67 – 50$

$$= 88.67 \text{ mm, say 90 mm < 100 mm, hence permissible}$$

b) Check for cant excess:

Equilibrium cant for goods train at 50 KMPH

$$= 8.32* \frac{V_g^2}{R}$$

$$= 8.32* \frac{50*50}{600}$$

$$= 34.67 \text{ mm}$$

Cant excess for the cant provided

$$= 90 \text{ mm} – 34.67 \text{ mm} = 55.33 \text{ mm, which is less than maximum permissible 65mm hence OK.}$$

**Hence adopt actual cant of 90 mm. [Ans]**
ii) Length of transition:
Since $C_a + C_d$ required for 100 KMPH = 138 mm,
$C_d = 138 – 90\text{mm} = 48\text{mm}$
From eqn (1.12), (1.13) and (1.14), the length of transition required is more than,

a) $L_1 = 0.008 \times C_a \times V_m = 0.008 \times 90 \times 100 = 72\text{m}$ or
b) $L_2 = 0.008 \times C_d \times V_m = 0.008 \times 48 \times 100 = 38.4\text{m}$, or

$c) L_3 = 0.72 C_a = 0.72 \times 90 = 64.8\text{m}$

Therefore provide transition length of 80 m or 8 station units.[Ans]

iii) If the transition is limited to 40m:
Since available length of transition is limited to 40m, which is less than the minimum length of 72 m, the cant and cant deficiency values have to be restricted$^{10}$. For finding the optimum values of cant and cant deficiency, the expression for speed on the transition curve should be equated to the expression for speed on the circular curve.

Further, for optimizing the length of the transition, cant and cant deficiency should be equal, i.e. $C_a = C_d$.

Transposing equation (1.12), $V_m = \frac{r_{cd} \times L}{C_a}$

Here $V_m$ is in m/sec. To convert the same into KMPH, a factor of 3.6 is to be used, i.e. $V_m = \frac{r_{cd} \times L}{C_a} \times 3.6$ ...................(3.12)

Also, for MG $V_m = 0.347 \sqrt{C_a + C_d}$ .............(3.13)

Since the length of transition is restricted, the limiting value of $r_{cd}$ i.e. 35 mm/sec shall be used. Substituting $C_a = C_d$, and equating the two values of $V_m$ from equations (3.12) and (3.13), we get

$$\frac{35 \times 40}{C_a} \times 3.6 = 0.347 \sqrt{2C_a} \times \frac{600}{C_a}$$

$^{10}$ See para 1.8.9, Chapter I.
\[ C_a = \left[ \frac{(35)^2 \times (40)^2 \times (3.6)^2}{(0.347)^2 \times 2 \times 600} \right]^{1/3} \]

\[ C_a = 56.02 \text{ mm} \]
i.e. \( C_a = C_d = 56.02 \text{ mm} \).

Since the value of \( C_d \) is more than the limiting value, the same is taken as 50 mm.

The actual cant to be provided is rounded off to 55 mm.

Cant gradient for 55 mm cant and length of transition of 40m is 55 mm/ 40 m, i.e. 1 in 727.27. This is flatter than maximum permissible 1 in 720, hence O.K.

Hence \( V_m = 0.347 \sqrt{(55 + 50) \times 600} \)

\[ = 87.10 \text{ KMPH rounded off as 85 KMPH.} \]

Therefore, maximum permissible speed on the curve when the transition length is restricted to 40 m is 85 KMPH.[Ans].

**Example 3.15**

On a double line suburban section (maximum permissible speed 100 KMPH) the track centers are 5 m apart. An island platform of 10m width is to be provided. One of the lines is to be diverted to accommodate the platform by introducing reverse curves on either side of the platform. Find out the optimum radius of the reverse curve, so as to keep the total length of the reverse curve minimum. Also find the cant and length of transition to be provided at the ends of the reverse curve as well as at the junction of the circular curves.

If the room available for provision of the full length of the reverse curve including transition is 120 m only, find the radius, cant and maximum permissible speed. The limiting values of cant deficiency, rate of change of cant, rate of change of cant deficiency, rate of change of cant, rate of change of cant deficiency and cant gradient are 75mm, 55mm/sec, 55mm/sec, and 1 in 360 respectively.
Solution:

![Diagram 3.3](image)

**Fig. 3.3**

i) When there is no restriction on availability of space for the diversion:

As calculated in example 3.1, a minimum radius of 640 m is needed for a maximum permissible speed of 100 KMPH. However on a diversion, this radius would involve more cant and hence lengthy transitions and will require more space. On the other hand, if the radius is kept larger than 640m, then also it would require more space on account of the longer length of the circular portion of reverse curves. Hence it is a problem of optimization of both radius and required transition length for that radius, while permitting the maximum permissible speed. For the optimal solution, we can proceed as follows:

![Diagram 3.4](image)

**Fig. 3.4**

Total length of the reverse curve is HN’ in Figure 3.4 above.

To compute the same, let L be the length of transition for a superelevation $C_a$ for a speed of 100 KMPH on a curve of radius R. Then the length of straight CD on the common tangent between the reverse curves without any transition in between is also equal to L.
Further, let HA = AJ = MG = GN = M'G' = G'N' = L/2

Length of common transition (KP) of the transitioned reverse curve is equal to 2L for equal C_a and R on the reverse curve.

Also KC = CE = ED = DP = L/2

Now, the total space required for the reverse curve including transitions.\(^{11}\)

\[
= HA + AG' + G'N' = L/2 + \sqrt{CD^2 + 4Rd - d^2} + L/2
\]

This expression indicates that the space required, depends on two variables viz. L and R

If C_a = C_d, the length of diversion will be optimum. For full sectional speed, 100 KMPH, the minimum length of diversion will be when C_a = C_d = 75mm.

Using eqn. (1.6), we get

\[
C_a + C_d = 75 + 75 = 13.76 * V^2/R
\]

Or \[
R = \frac{13.76 * 100 * 100}{150} = 917.33m \text{ say } 920m
\]

Length of transition required shall be more than:

\[
L_1 = L_2 = 0.72 \times C_a = 0.72 \times 75 = 54 \text{ m}
\]

Or \[
L_3 = 0.008 \times (C_a \text{ or } C_d) \times V_m
\]

= 0.008 \times 75 \times 100 = 60m.

Adopt L = 60 m for C_a = C_d = 75 mm and R = 920 m \[\text{[Ans.]}\]

The space required for diversion is given by

\[
HN = HA + G'N' + AG'
\]

\(^{11}\) See para 712(4) of IRPWM.
i.e. \( HN = L/2 + L/2 + \sqrt{4 \cdot Rd - d^2 + L^2} \)

\[ = 30 + 30 + \sqrt{4 \times 920.00 \times 8.352 - (8.352)^2 + 60^2} \]

\[ = 245.11 \text{ m say 245 m.} \]

Length of Transition at ends = 60m.
Length of Transition at junction of reverse curves = 120m
Total length of diversion = 245m \( \text{[Ans]} \)

(ii) When spare available for provision of full reverse curve including transition is limited to 120 m as against 245 m, it will be necessary to adopt a sharper curve, as well as a short transition for minimizing the required space.

This will naturally require a speed restriction.

We know that \( HN' = 120 \) m.

Adopting a minimum length of transition as 15 m (so as not to be less than the minimal virtual transition of 14.60 m for BG.) we get

\[ 120 = \frac{L}{2} + \sqrt{4 \cdot Rd - d^2 + L^2} + \frac{L}{2} \]

\[ \therefore 120 = \frac{15}{2} + \sqrt{4 \cdot R \times 8.352 - 8.352^2 + 15^2} + \frac{15}{2} \]

Solving for \( R \), we get

\[ R = 325.364 \text{ m} \]

Adopting a radius of 325 m for the reverse curves, total length required

\[ HN = \frac{15}{2} + \sqrt{4 \times 325 \times 8.325 - (8.352)^2 + 225 + \frac{15}{2}} \]

\[ = 119.942 \text{ m say 120 m.} \]

Proceeding on similar lines as in example 3.5 to find out permissible speed on curve,
\[ V_m = \frac{\text{Rate of change of Cant} \times \text{Length of transition}}{C_a} \times 3.6 \]

\[ = 0.27 \sqrt{(C_a + C_d)R} \]

For optimum length of diversion, \( C_a = C_d \),

\[ \frac{55 \times 15 \times 3.6}{C_a} = 0.27 \times \sqrt{2C_a \times 325} \]

\[ C_a = \left[ \frac{(55)^2 \times (15)^2 \times (3.6)^2}{(0.27)^2 \times 2 \times 325} \right]^{1/3} \]

\[ C_a = C_d = 57.1 \text{ mm} \]

However, from cant gradient consideration maximum \( C_a \) is limited to \( (15 \times 1000)/360 \) i.e. 41.66 mm

Hence adopt, \( C_a = 40 \text{ mm} \)

Therefore, \( V_m = 0.27 \sqrt{(40 + 57.1) \times 325} \)

\[ = 47.96 \text{ KMPH, rounded off to 45 KMPH.} \]

Hence adopt the following values for the relevant parameters in case of limited availability of room for diversion :-

Radius of the reverse curve = 325 m
Actual cant, \( C_a = 40 \text{ mm} \)
Cant deficiency, \( C_d = 57.1 \text{ mm} \)
Length of transition, \( L = 15 \text{ m} \)

Maximum permissible speed = 45 KMPH.

[Ans]

**Example 3.16**

A curve of radius 825m is to be set between two straights with the deviation angle of 60°. The maximum permissible speed is 100KMPH. The speed of the goods trains is 75KMPH. The
equilibrium speed of the section as decided by the Chief Engineer is 80 KMPH. Find the location of the tangent point. Also calculate the offsets of the transition curve. If the chainage of the starting point of curve is 253051, find out the chainage of the end of curve.

**Solution:**

The first step is to calculate the $C_a$. Since the equilibrium speed is given, this can be calculated as follows

\[
C_{eq} = 13.76 \times \frac{V_{eq}^2}{R}
\]

\[
= 13.76 \times \frac{80^2}{825}
\]

\[
= 106.74 \text{mm}
\]

say 105mm

Using eqn (1.6), Cant deficiency,

\[
C_d = 13.76 \frac{V_m^2}{R} - C_{eq} = \frac{13.76 \times 100^2}{825} - 105
\]

\[
= 61.79 \text{mm} < 75 \text{ mm permitted, hence OK.}
\]

Using eqns. (1.12), (1.13) and (1.14), the length of the transition curve is calculated as maximum of the following three values

\[
L_1 = 0.008 \times C_a \times V_{max} = 0.008 \times 105 \times 100 = 84m
\]

\[
L_2 = 0.008 \times C_d \times V_{max} = 0.008 \times 61.79 \times 100 = 49.4m
\]

\[
L_3 = 0.72 \times C_a = 0.72 \times 105 = 75.6m
\]

Select $L = 84m$ say 90m (rounding off to next 10m)

The value of shift is calculated as per equation (1.21),

\[
\text{Shift } S = \frac{L^2}{24R}
\]

\[
= \frac{90 \times 90}{(24 \times 825)}
\]

\[
= 0.409m
\]

For setting out the curve, we use principles given in Chapter IV. When the shift is introduced, the circular curve shifts inside and the distance OF becomes 825.409m (refer to figure 3.5)
Fig. 3.5

Now in figure 3.5, in Δ OFX, FX = 825.409 x tan 30°
= 476.550m
Transition curve is provided half in straight and half in circular curve, so AF = ½ of transition length
= 90/2 = 45m

Hence AX = AF + FX
= 45.0 + 476.550
= 521.550m

i.e., point ‘A’ can be located by measuring a distance of 521.550 m from point ‘X’ along the rear on first tangent. [Ans.]

Using equation (4.5), the deflection angle for each transition portion in cubic parabola is

\[ \Phi_t = \tan^{-1}\left(\frac{L}{2R}\right) \]
\[ = \tan^{-1}\left(\frac{90^*}{2 \times 825}\right) \]
\[ = 3.122^\circ \]

Hence the deflection angle for circular curve
\[ = 60^\circ - 2 \times 3.122^\circ \]
\[ = 53.756^\circ \]

The length of the shifted circular curve TC - CT between the two transitions (using eqn. (4.6)) is
\[ L_{\text{cir}} = \pi \times R \left( \Delta - 2\phi_t \right)/180^\circ \]
\[ L_{\text{cir}} = \pi \times 825 \times 53.756^\circ /180^\circ \]
\[ = 774.03 \text{m} \]

The equation of transition curve is as per equation (4.2) is
\[ Y = x^3/6RL \]

Substituting the values of \( x \) as 10, 20, 30 etc., the values of offset \( Y \) can be calculated and are listed below:

<table>
<thead>
<tr>
<th>( x )</th>
<th>( Y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>0.0022m</td>
</tr>
<tr>
<td>20</td>
<td>0.0180m</td>
</tr>
<tr>
<td>30</td>
<td>0.0606m</td>
</tr>
<tr>
<td>40</td>
<td>0.1436m</td>
</tr>
<tr>
<td>50</td>
<td>0.2805m</td>
</tr>
<tr>
<td>60</td>
<td>0.4848m</td>
</tr>
<tr>
<td>70</td>
<td>0.7699m</td>
</tr>
<tr>
<td>80</td>
<td>1.1492m</td>
</tr>
<tr>
<td>90</td>
<td>1.6364m</td>
</tr>
</tbody>
</table>

[Ans.]

This curve can also be set using tangential angle. For this purpose a theodolite is set at point ‘A’ (refer to fig 3.5). Select a chord length of 20m. The tangential angle is calculated as
\[ \tan \alpha = x^2 / (6RL) \]
\[ \alpha = \tan^{-1} \left( x^2 / 6RL \right) \]

Substituting the values of \( x \) as 20, 40 – etc., the values of offset can be calculated and are listed below:
<table>
<thead>
<tr>
<th>x</th>
<th>α</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>20</td>
<td>0° 3' 5.2&quot;</td>
</tr>
<tr>
<td>40</td>
<td>0°12' 20.8&quot;</td>
</tr>
<tr>
<td>60</td>
<td>0° 27' 46.7&quot;</td>
</tr>
<tr>
<td>80</td>
<td>0°49' 23&quot;</td>
</tr>
<tr>
<td>90</td>
<td>1° 2' 29.86&quot;</td>
</tr>
</tbody>
</table>

Since the chainage of the starting point of curve is 253051, chainage at end of curve will be:
\[ = 253051 + 90.0 + 774.04 + 90.0 \]
\[ = 254005.04 \]

The above data can be used for setting out the curve as per principles given in Chapter 4.

Example 3.17
In group ‘A’ route (B.G.), a rising gradient of 1 in 100 meets a falling gradient of 1 in 200 at a point of intersection whose chainage is 1000.00m and whose RL is 100.00m.
Is a vertical curve required to be provided? If so, set out the vertical curve between the two gradients.

Solution:

The algebraic difference of gradients
\[ = \frac{1}{100} - \frac{1}{200} = \frac{1}{100} = \frac{3}{200} = \frac{15}{1000} \]

This is equivalent to 15mm per metre, which is more than 4 mm per metre\(^{12}\). So a vertical curve has to be provided.

[Ans.] Since the route is A, the radius of the vertical curve, to be provided is 4000m.

\(^{12}\) See para 1.12, Chapter I.
Tan $\theta_1 = 1/100$

For small values of $\theta_1 = \tan \theta_1 = \theta_1 = \text{in Radians}$

Therefore $\theta_1 = 1/100$ Radians

Similarly $\theta_2 = 1/200$ Radians

Therefore Deflection angle $\theta$ at $O = \theta_1 + \theta_2$

$= 1/100 + 1/2000 = 15 / 1000$ Radians

Since the value of deflection angle is very small, the following assumptions can be made which will enable quicker calculations for setting out of the curve, without loss of accuracy:

1. Length of the vertical curve $ABC = \text{Length of chord } AB'C = R\theta$

   $= 4000 \times \frac{15}{1000} = 60\text{m}$

2. Tangent length $AO$  \( R\tan \frac{\theta}{2} = R\frac{\theta}{2} \)

   $= 4000 \times \frac{15}{1000} \times \frac{1}{2} = 30\text{m}$

Now, chainage of point $A = 1000 - 30 = 970\text{m}$

$\therefore RL_A = 100 - 30 = 99.70\text{m}$
Also, chainage at B = 1000 + 30
= 1030m

:\ RL_B = 100 - \frac{30}{200} = 99.85m

(3) The RL of any point say N on the vertical curve is found by summation of RL of the point N on the chord and the normal offset NN' between the curve and its chord. For convenience of setting out the vertical curve, the chord length ABC is equally divided into a convenient even number of subchords. In the present case, since the chord length is 60m, it is divided into 6 equal parts as M', N', B', S' and T'. Offsets from chord can be calculated using equation (1.2) for versine:

\[ BB' = \frac{C^2}{8R} = \frac{60^2}{8 \times 4000} = 0.1125m \]

\[ NN' = SS' = BB' - BL = 0.1125 - \frac{20^2}{8} \times 4000 = 0.1m \]

\[ MM' = TT = BB' - BK = 0.1125 - \left(\frac{40^2}{8}\right) \times 4000 = 0.0625m \]

The chainages of points M', N', etc., on the chord are assumed to be the same as chainage of point M, N etc., on the curve. Therefore Chainages of M, N etc. are as follows:

<table>
<thead>
<tr>
<th>Point</th>
<th>Chainage</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>970.00</td>
</tr>
<tr>
<td>M, M'</td>
<td>980.00</td>
</tr>
<tr>
<td>N, N'</td>
<td>990.00</td>
</tr>
<tr>
<td>B, B'</td>
<td>1000.00</td>
</tr>
<tr>
<td>S, S'</td>
<td>1010.00</td>
</tr>
<tr>
<td>T, T'</td>
<td>1020.00</td>
</tr>
<tr>
<td>C</td>
<td>1030.00</td>
</tr>
</tbody>
</table>

The levels of A and C are known, so reduced levels of the points M', N' etc., on the chord can be interpolated as follows:
Now we know R.L.s of M, N, etc. and offsets MM', NN' etc.

Therefore, R.L.s of points M, N, etc. on vertical curves are:

<table>
<thead>
<tr>
<th>Point</th>
<th>Chainage</th>
<th>Reduced Level (RL)</th>
</tr>
</thead>
<tbody>
<tr>
<td>M</td>
<td>980.00</td>
<td>RL of M' + MM' = 99.725 + 0.0625 = 99.7875m</td>
</tr>
<tr>
<td>N</td>
<td>990.00</td>
<td>RL of N' + NN' = 99.75 + 0.10 = 99.8500m</td>
</tr>
<tr>
<td>B</td>
<td>1000.00</td>
<td>RL of B' + BB' = 99.775 + 0.1125 = 99.8875m</td>
</tr>
<tr>
<td>S</td>
<td>1010.00</td>
<td>RL of S' + SS' = 99.80 + 0.1 = 99.9000m</td>
</tr>
<tr>
<td>T</td>
<td>1020.00</td>
<td>RL of T' + TT' = 99.825 + 0.0625 = 99.8875m</td>
</tr>
</tbody>
</table>

Using above chainages and RLs, the vertical curve can be set out.
3.4 CHAPTER III REVISION QUESTIONS

1. On a curve of radius 1000 m on Rajdhani route (maximum speed 130 KMPH), find out the cant, permissible speed and desirable length of transition assuming booked speed of goods train as 65 KMPH.

2. If transition length is limited to 50 m in a BG group D route curved track of radius 875 m, find out the speed potential of the curve.

3. Find out the equilibrium speed by Russian formula for a typical section with the following trains:

<table>
<thead>
<tr>
<th>S No</th>
<th>Type of Train</th>
<th>Nos (n)</th>
<th>Average Load (Tonnes)</th>
<th>Average Speed (KMPH)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Rajdhani</td>
<td>2</td>
<td>1700</td>
<td>130</td>
</tr>
<tr>
<td>2</td>
<td>Mail/Express</td>
<td>15</td>
<td>1400</td>
<td>110</td>
</tr>
<tr>
<td>3</td>
<td>Passenger</td>
<td>5</td>
<td>1100</td>
<td>100</td>
</tr>
<tr>
<td>4</td>
<td>Main line EMU</td>
<td>5</td>
<td>900</td>
<td>80</td>
</tr>
<tr>
<td>5</td>
<td>Container</td>
<td>6</td>
<td>2800</td>
<td>100</td>
</tr>
<tr>
<td>6</td>
<td>Loaded Goods</td>
<td>4</td>
<td>4700</td>
<td>60</td>
</tr>
<tr>
<td>7</td>
<td>Empty Goods</td>
<td>2</td>
<td>2200</td>
<td>70</td>
</tr>
</tbody>
</table>

4. An MG curve of 2 degrees is to be converted to BG. The existing curve had a transition of 50 m length. If the sectional speed in BG is to be 100 KMPH, find out the maximum speed potential and desirable length of transition. Also find out the shift of track required.

(Hint: Find out shift of existing track and that for the new track. The difference is the amount by which the track is to be shifted)

5. Speed on a section is to be raised from 130 KMPH to 160 KMPH. Find out the increase in length of transition in a curve having radius 2000 m.
CHAPTER IV

SETTING OUT OF CURVES
CHAPTER IV

SETTING OUT OF CURVES

4.1 Choosing a Curve:

Whenever there is any obstruction in the path of a straight track or there is some obligation (See para 1.1, Chapter I), the path of the track has to be changed by introduction of a curve. In order to have smooth running between the two directions of the track desired, the curve shall be laid such that it is tangential to the both the tangent tracks. One such instance where a water body is causing obstruction is seen in figure 4.1 below. (The figure is drawn as bird’s eye view.)

![Fig. 4.1](image)

Now, these are various options of curves between same set of tangent tracks. (We are considering only circular curves at the moment, transition will be fitted later on.)

The option I is decided by the consideration of minimum radius of curve based on the speed potential of the track being laid, i.e. curve having radius less than certain value cannot be laid otherwise there will be a Permanent Speed
Restriction (PSR). The option I in figure 4.1 is not feasible as it is cutting right through the water body which had prompted the introduction of curve.

The option II is feasible as it leaves sufficient margin by the side of the water body. However, it may be seen that the length of curved track is quite large in option II.

Therefore, the extra distance from water body as seen in figure 4.1 is not good and optimum curve shall be one that passes at the least required distance away from the water body, as seen in the figure.

Other instances where the curves are required to be introduced may be seen in figures 4.2 and 4.3 below. The figure 4.2 shows plan of a set of reverse curves where some work such as rebuilding of bridge is being done for which the track is to be side shifted. Figure 4.3 shows plan of a track layout where the direction of tracks to be changed to meet a population center which is an obligatory point.

Fig. 4.2: Curves introduced for diversion for providing a new bridge
Fig. 4.3: Curves introduced for meeting obligatory point

From the above discussion, it is seen that the following guidelines for choosing radius of the curve may be followed in field:

- Curve has to be chosen along feasible alignment i.e. the land shall be free and the construction of the track on the same shall be feasible.
- Radius less than a specified value cannot be chosen unless there are very strong technical or economic reasons which will justify the permanent speed restriction or reduced speed potential on the curve.
- The radius chosen will also be governed to some extent by the grade compensation considerations, if the location of the curve also happens to be on an up gradient, as discussed in para 2.14, chapter II. If some gradient is there at the location of the curve, the minimum curve radius shall be increased to compensate for the gradient.
- Too slack radius cannot be chosen as it increases the requirement of land.
4.2 Elements of a Curve: A curve can be set out in field only if we know its elements. Various elements of a curve are shown in figure 4.4. The curve (circular as well as transition at either end) is fitted between the two tangents as shown earlier and the angle between the two tangents is $\Delta$ at the point of intersection of the two tangents, $I$. Now, $ST$ is the point from which the transition curve starts from the straight. First of all, the transition of length $L$, shown as $\_\_\_\_\_\_\_\_\_\_\_\_$ is fitted in and the same extends between $ST$ and $TC$. The circular curve is between the points $TC$ and $CT$. The circular curve is shown in solid line ———. At the end of the circular curve, the other transition is provided, between $CT$ and $TS$. Beyond $TS$, the second straight is there.

For computation purpose, we consider a circular curve, shown in figure 4.4 and again drawn in figure 4.5, which is known as the equivalent circular curve. This curve starts from the point $T_1$ and ends at point $T_2$ and the length of the same is $CL$. 
The various elements of the curve shown in figure 4.4 are:

- Deflection angle, $\Delta$
- Radius, $R$
- Tangent length, TL
- Transition length, L
- Overall length of curve, OAL (From ST to TC to CT to TS)
- Length of equivalent circular curve, CL (From $T_1$ to $T_2$ in figure 4.5)

4.3 Determining the Elements of a Curve:

4.3.1 Deflection angle, $\Delta$:

- The general alignment that the track shall take is known from the starting point and the end stations which are to be joined together. The obligatory points change the general alignment of track. Other considerations such as ruling gradient also affect the alignment, which are considered while carrying out preliminary survey.

- During preliminary survey, two – three alternative routes are studied. The track is plotted on the contour sheets to get an idea of the gradients. The general features of the soil, topography, built up areas, revenue likely to be earned, engineering problems likely to be encountered are also studied along with some elementary data collection from the actual site. If required, the direction of the track is changed to avoid obstructions, bad geological formations etc already known.

- From amongst the various options, one alignment which is chosen is taken up for detailed survey. The actual site data is collected along with the details of land to be acquired. The final alignment showing the general direction of tangent tracks is now decided taking into account the actual site conditions.

- The change in the direction of alignment will give the deflection angle at the junction of the straights. In figure 4.4/4.5 this is denoted by $\Delta$. 
4.3.2 Radius of curve (R):

- Let us consider the figure 4.1 again. To clear the obstruction, the radius of the curve is governed by the apex distance.

- As seen in the figure 4.5, the apex distance also known as versed sine is equal to
  \[ R \times \sec \frac{\Delta}{2} - R \] ..........................(4.1)

- Since \( \Delta \) is already known from 4.3.1 above, and we know the minimum apex distance which will clear the obstruction, we can find the minimum radius of curve which can be provided.

4.3.3 Other Main Parameters: Now that the \( R \) and \( \Delta \) are known, the other parameters, which are required for plotting the curves can be determined.

4.3.3.1 Length of equivalent circular curve, CL: Let us see figure 4.5. In the figure, \( T_1 \) is the first tangent point from where the circular curve takes off from the straight (or tangent), \( T_2 \) is the second tangent point, where the circular curve meets the
second straight (or tangent). I is the apex point and the alignment would have met at I, had the circular curve not set out between them. The curve length between $T_1$ and $T_2$ is given by the formula curve length, $CL = 2\pi R \Delta/360$.

**4.3.3.2 Length of circular curve, CCL:** The circular curve as seen in the figure 4.5 is not very good curve from railway point of view as it suddenly changes the curvature and limited amount of superelevation can be provided in such curves, as discussed in paras 1.5, 1.8 and 1.11, Chapter I. In most of the cases in field, a transition curve is inserted at either end of the circular curve. The elements for the combined curve are shown in figure 4.6. Superimposing the figures 4.4 and 4.5, we get figure 4.6 below.

Here, it is seen that as discussed in para 1.11, chapter I, the circular curve shifts inside to accommodate the transition curves. A transition of length, L, has been inserted between the straight and the circular curve. (The length of transition in this case has been assumed equal, however it can also be different at the two ends.) As discussed in para 1.8, Chapter I, the transition curve is accommodated half in the straight and
half in the circular curve, therefore the beginning of curve shifts towards the straight, by an amount $L/2$. The circular curve shifts inwards by an amount $S$ given by eqn (1.21).

From equation (1.11), the equation of a transition is:

$$y = \frac{x^3}{6RL} \quad \text{..............................................................................................} \quad (4.2)$$

Now, deviation angle from tangent to any point on transition, (see figure 4.7)

$$\tan \alpha \approx \frac{y}{x} = \frac{x^2}{6RL} \quad \text{..............................................................................................} \quad (4.3)$$

Where $x$ is measured from the point ST and $\alpha$ is calculated for each station from the original tangent.

And deviation angle for the complete transition, $\alpha_t$, for $x = L$ is

$$\alpha_t = \frac{L^2}{6RL} = \frac{L}{6R} \quad \text{..............................................................................................} \quad (4.4)$$

The deflection angle at any point on curve is the angle made by tangent at that point on curve with original tangent at beginning of curve and is denoted by $\tan \phi = \frac{dy}{dx}$

Since $\phi$ is small, $\tan \phi \approx \phi$

For transition curve $y = \frac{x^3}{6RL}$;

Deflection angle at any point $(x, y)$

$$\tan \phi \approx \phi = \frac{dy}{dx} = \frac{3x^2}{6RL} = \frac{x^2}{2RL} \quad \text{.................................} \quad (4.5)$$

At the end of transition where $x = L$

Deflection angle $\phi_t = \frac{L^2}{2RL} = \frac{L}{2R}$

It can be seen from equation (4.3) and (4.5) that for transition curve, deviation angle at any point: $\alpha = \frac{1}{3}$ of deflection angle $\phi$
Similarly Deviation angle for the complete transition, $\alpha_r = \frac{\phi_r}{3}$

Now, due to insertion of transitions at either end, some deflection has already occurred, so the deflection angle of the circular curve reduces from the overall deflection angle. The length of Circular curve also reduces correspondingly and is given by:

$$CCL = \frac{2\pi R}{360}(\Delta - 2\phi_r)$$  \hspace{1cm} (4.6)

4.3.3.3 Chainages of various points:

Chainage of ST: We already know the chainage of the apex point from the initial survey when we fixed up the alignment of the track. Using geometry, in figure 4.6, the total tangent length,

$$l - ST = (R + S) \tan \frac{\Delta}{2} + \frac{L}{2}$$

Therefore, the chainage of ST=

$$(R + S) \tan \frac{\Delta}{2} + \frac{L}{2}$$  \hspace{1cm} (4.7)

Chainage of TC: Chainage of end of transition curve,

$$TC = \text{Chainage at ST} + \text{length of transition}$$  \hspace{1cm} (4.8)

Chainage of CT: Chainage of end of circular curve,

$$CT = \text{Chainage at TC} + \text{Length of circular curve},$$

$$\hspace{1cm} CCL ....(4.9)$$

Chainage of TS: Chainage of end of curve,

$$TS = \text{Chainage at CT} + \text{Length of transition},$$

$$\hspace{1cm} L .......(4.10)$$

Total Length of curve:

$$OAL = CCL + 2 * L$$  \hspace{1cm} (4.11)

4.3.4 Calculations For setting Out a Curve:

In order to set out a curve, we need to fix the position of stations at every 10 m or so. In the transition portion since direction changes continuously, the stations may be set out at lesser distance. For each station, deviation angle from the tangent also needs to be determined.
For the transition portion, the deflection angle is given by the eqn (4.5) above. If \( x_1 \) is the distance from the ST to the first station, the deflection for the first station (see figure 4.7) will be:

\[
\phi_1 = \frac{x_1^2}{2RL} \quad \text{and} \quad \alpha_1 = \frac{\phi_1}{3} \quad \text{..................................................(4.12)}
\]

For subsequent stations at distance \( x_2, x_3, \ldots \) from the previous stations, the deflection angle from original tangent will be

\[
\phi_2 = \phi_1 + \frac{x_2^2}{2RL}, \quad \phi_3 = \phi_2 + \frac{x_3^2}{2RL} \quad \text{..................................................(4.13)}
\]

The deviation angle at each of these points then can be calculated as \( \alpha_i = \frac{\phi_i}{3} \).

For circular curve, by simple geometry, if the chord length (distance between the stations being set up) is \( c \), the deflection angle, \( \delta = \frac{360}{2\pi R} \ast c \) ................................................. (4.14)

If the distance of the first station from the TC point is \( c_1 \), the deflection angle from the tangent at TC will be
\[ \delta_1 = \frac{360}{2\pi R} \cdot c_1 \] ..............................................................(4.15)

If further stations in the circular curve are at distances \( c_2, c_3 \) and so on, the deflection angles from straight will be:

\[ \delta_2 = \delta_1 + \frac{360}{2\pi R} \cdot c_2; \quad \delta_3 = \delta_2 + \frac{360}{2\pi R} \cdot c_3 \text{ and so on} ..........(4.16) \]

As a check, the total deflection angle for the circular portion of the curve, between the points TC and CT will be \( \Sigma \delta_i = \frac{360}{2\pi R} \cdot \text{CCL} \).

For setting of circular curve, we need deviation angles. In circular curve the relation between deflection and deviation angle is as under:

\[ \text{deviation angle } \gamma_i = \frac{\text{deflection angle } \delta_i}{2} \] .............................................(4.17)

**Note:**

1) *For the other transition, the calculations can be done from the other tangent point* i.e. *TS in similar fashion as above.*

2) *Even for the circular curve, the calculations may be done partly from one side and partly from the other side.*

3) *Normally, except for the first chord and the last chord, all other chords are kept equal to 10 m and separate calculations are not required to be done for each point.*

4) *The deviation angle, gamma, for each station is half the values of delta worked out as above , and can be used to set out the stations on circular part of a curve.*

### 4.4 Setting out a Curve

#### 4.4.1 Using Tape and Theodolite:

The steps for setting out curve by this method are:

**Transition portion:**

a) Locate points ST on the first tangent and TS on the second tangent using the equation (4.7) to (4.10) and using the chainage readings from the apex I, which is already known from the final location survey.
b) Set theodolite at ST, set versine to zero, clamp the upper screw.

c) Sight apex point, I and clamp the lower screw. If the apex point is inaccessible, sight some other point on the tangent either in forward direction (I') or in the reverse direction (I'') (see figure 4.7). If the point I' is sighted, proceed as below. If the point I'' is sighted, the theodolite has to be plunged (telescope rotated about the horizontal axis) after sighting so that the theodolite is pointing in the direction of I.

d) Release upper screw, swing the theodolite by the first deviation angle $\alpha_1$ calculated using equation (4.12).

e) Take the tape opened by an amount equal to $x_1$. The tape is swung till the stadia hair of the theodolite intersects the tape. Peg this point, $X_1$.

f) Open the tape equal to $x_2$ from previous station, release the upper screw and set the theodolite such that the deviation angle now reads $\alpha_2$ calculated using equation (4.13). Swing the tape as described in para 4.4.1 (e)

g) Similarly peg all the points on the transition upto TC.

Circular Portion:

h) Now shift the theodolite to TC. Clamp the horizontal angle in theodolite at $360^\circ - 2*\alpha_i$, telescope pointed towards the point ST. Plunge the telescope and swing the theodolite by angle $2*\alpha_i$ (See figure 4.8). Now the telescope is pointing along the tangent at TC with reading in the horizontal scale showing zero. The zero reading in horizontal circle of theodolite will simplify calculations for angles for all the points subsequently.

i) Mark the points on the circular curve using the same procedure as described in paras d) to f) above using the $\delta_i$ and $c_i$ values calculated with the equations (4.15) and (4.16) and deviation angle $\gamma$ may be calculated from equation (4.17). As seen in figure 4.8, the pegging stations on circular curves done by swinging tape the point pegged every time. The circular curve upto the point CT is pegged in similar manner.
j) Using the procedure outlined above, the other transition can be pegged by similar procedure from the tangent point TS. If the curve is long, half of the curve can be pegged from one tangent and other half from the other tangent.

4.4.2 Setting out Curve by two theodolite method: The setting out of curve can be done using two theodolites, eliminating the need for using tape completely. This procedure is quite useful when the ground is undulating.

a) The deflection angle for all the points shall be determined as outlined in para 4.3.4. The deflection angles shall be found out from one tangent for all the points.

b) Set out the transition curves as explained earlier by using tape and theodolite or by methods described further in para 4.4.1 from ST to TC and from TS to CT.

c) As seen in figure 4.8, the total deflection angle for the transition at the tangent is $3\alpha_t$. Therefore, after setting out the two transition curves, the angle for the circular curve at the center of the curve, O is: $\Delta' = \Delta - 2 \times 3 \times \alpha_t$.

d) Follow the procedure explained in para 4.4.1 i) above and shift the two theodolites to the points TC and CT and align both along the tangents to the curve, as shown in figure 4.9 towards $I_1$ and $I_2$.
e) Now the circle has a property that “angle subtended by any chord at the center of the curve is twice the angle between the chord and tangent at an end of the chord”.

f) From equation (4.15), (4.16) and (4.17), we can work out the angles $\gamma_1$, $\gamma_2$ etc for the chords TC – A, A-B etc of the circular curve. Using the property defined in para e) above, we get the angles at the center of the circle as $2\gamma_1$, $2\gamma_2$ etc. And from the same property, we see that the angle $I_2$-CT-A shall be $\Delta'/2 - \gamma_1$ and the angle $I_2$-CT-B shall be $\Delta'/2 - \gamma_2$.

g) To set out the first point, A swing the theodolite set at ST horizontally by and angle $\gamma_1$ in the clockwise direction. Simultaneously, swing the theodolite set at TS horizontally by and angle $\Delta'/2 - \gamma_1$ in the anti-clockwise direction.
h) Move a ranging rod such that the point is at the intersection of the stadia hair in both the theodolites. The point so found is A.

i) Now, proceed further and stake the point B and so on using the similar procedure as above

4.4.3 Use of Modern Surveying Equipment and Computers:
The theodolite, tape and ranging rod are all old methods of setting out of curve. The modern surveying equipments such as Electronic Distance Measuring equipments (EDMs), Total station, and GPS based equipment are available in the market and are bring used widely for setting out track alignment including the curves.

4.4.3.1 Computations for a curve: Various computations have to be done for each station. The para 4.3.3 and 4.3.4 describe the methods of doing the calculations for the curve, which can be used further to find out the x-coordinates or northings and y-coordinates or eastings of the points on the curve. These computations can also be done using computers. Quite a few softwares are available in the market that give these values directly from the raw data taken for detailed survey from site.

4.4.3.2 Setting Out curves Using Total Station:

   a. To set out the curve, the calculated x and y coordinates of all the points are fed in the total station.

   b. Go to stake out function of total station. Set up the total station at the first point near tangent point and establish back site orientation from a bench mark. The total station with display the information of rotation angle (ΔH) and horizontal distance (ΔD) of point to be marked on ground from instrument position.

   c. Rotate the instrument till ΔH becomes 0°0'0".

   d. Align prism-man in line with the line of sight of the total station. Take first measurement.

   e. Total station calculates the distance by which prism-man is to be shifted forward or backward. Communicate this information to prism man who shifts accordingly.
f. Take another measurement. Repeat step (d) till the $\Delta D$ becomes zero and the point on the curve is then established.

g. Input next point number in total station and get new $\Delta H$ and $\Delta D$ values. Repeat steps (c) to (f) to establish the next point, and so on.

**Important Note:** In surveying, we shall always work from whole to part and it is desirable to establish control stations using precise traverse. These control stations shall be established every 250m or so and the further readings as listed out above shall be taken from these control points.

### 4.4.4 Setting out of small curves

#### 4.4.4.1 By ordinates from straight:

Small curves such as the turn in curves are better laid by ordinates from the straight as shown in figure 4.10.

Using equation (4.2) for the cubic spiral for transition portion, the ordinate from the straight shall be

$$ y = \frac{x^3}{6RL} $$

As per above, the ordinate at the end of the transition is:

$$ \frac{L^2}{6R} $$

Beyond the end of transition portion, the circular curve starts.

---

**Fig. 4.10**
The circular curves have a geometrical property that the ordinates from straight are $C^2/8*R$ (same as the versine).

Therefore the total ordinate, $Y_1$, shall be $\frac{L^2}{6R} + \frac{C^2}{2R}$ up to center of the circular curve. The ordinates beyond center can be set from the other end transition point and measuring in the reverse direction.

The x and y coordinates of the two ends of the curve, length of diversion and the angle $\phi$ are to be calculated manually or using computers.

4.4.4.2 By ordinates from the chord connecting the end points: Smaller curves can also be set out by ordinates from the chord connecting the two ends of the curve. Longer curves can be divided into parts, and each part can be separately set out as shown in figure 4.11. The simple computations in such a case are as follows:

Ordinate at x/2 from center = $(L^2/8*R)-(x^2/8*R)$

NOTE: There are many more cases in survey such as where the
tangent points are not visible or the apex point is not accessible/visible, where the curve is very long etc. However, the scope of this book is only to give a basic idea about setting out of curves to make the treatment of the subject complete. The cases not covered in this book may be referred to in specialized literature on surveying.

A computer program for curve calculations has been developed by Sh. M.S. Ekbote, Retd AMCE, Rly. Bd which has a module which calculates the various values of the parameters for setting out curves and diversions. This program is available in members’ download area in IRICEN website http://www.iricen.gov.in
CHAPTER V

REALIGNMENT OF CURVES
CHAPTER V

REALIGNMENT OF CURVES

5.1 Difficulty in maintenance of Curved alignment: The curved alignment differs from straight track in one important aspect i.e. the presence of lateral forces as centrifugal force, or centripetal force, depending on the speed of train on the curve. The lateral forces cause wear and tear of the rail, sleepers and fastenings and also cause the track geometry to deteriorate. Therefore the curved alignment requires more maintenance compared to the straight track.

The aim of permanent way engineers while maintaining curved track is to ensure proper versines, gauge and cross-level. It has been discussed in Chapter I that abrupt changes in curvature and super-elevation on curves will result in poor riding comfort on the curves, increased wear and tear of rails, fittings and wheels, and it may even be unsafe for the vehicles. The curves shall be maintained in such a manner that the versines and superelevation are varying gently and the super-elevation at each point should be appropriate for the curvature at that point.

5.2 Rectification of parameters in curve: As discussed above, the most important parameters of track in the curve to be monitored are: Gauge, Cross level and Versines.

a) The permitted limits of gauge on the curved track have been brought out in para 2.6, Chapter II. The procedure for rectification of gauge in curved track is similar to the procedure in case of straight track i.e. by making adjustments in the liners of the concrete sleepers. However, the gauge correction in the curves becomes slightly more difficult due to the wear in the gauge face of the outer rails. Beyond a certain limit, it will not be possible to rectify the gauge, and rails must be replaced or interchanged in this condition.

b) Superelevation: When the superelevation in curved track gets disturbed, the lower rail is taken as reference and the outer rail is raised as required. However, while rectifying the cross
levels, the vertical profile of the track is also to be kept in mind. The lowering of any rail for the rectification of the superelevation defects is normally not done.

c) Versines: As discussed in para 1.3 and 1.9, Chapter I, versine is the parameter used for the measurement and rectification of the curve geometry. The measurement of versines has been explained in para 2.2, Chapter II. The versines are measured on shifting chord. This means that each point is involved in versine reading of three stations and therefore, the shifting of any point on the curve will affect the versine readings at three stations. Therefore, while rectifying the defects in the versine readings, we have to keep the effect of changing the alignment at one point on the adjoining two points in mind. Failure to do so will only result in shifting the problem to another point nearby and the desired objective of alignment rectification will not be fulfilled. Due to this reason, rectification of versines is the most difficult part of curve maintenance.

The rectification of alignment of curve is called realignment of curves. The various cases necessitating realignment in field are:

   a. Local adjustment of curve: Quite often, in field, the curve gets disturbed in small stretches due to presence of special features in track such as the level crossings, points and crossing, bridge, SEJ etc. In such a situation, it is not required to attend to the entire curve and the rectification can be done in affected length(s) only. This rectification of alignment locally is called local adjustment of curve.

   b. Attention to transitions: The versine and superelevation values change continuously in transition portions. Due to constant change of curvature, lateral forces in the transition portion are also variable. Therefore, even slight disturbance of geometry in transition portions will lead to fast deterioration of geometry. The transition portions of the curves are, therefore, required to be attended more frequently. This rectification of geometry in the transition portion of the curves alone is called attention to transitions.

   c. Complete realignment of curve: When the curve is disturbed over a larger length and rectification of versine
readings can be done only after considering the curve as a whole, the rectification of alignment of curve is called **complete realignment of curve**.

**d. Increase in Transition length:** If there is a speed restriction on account of the inadequate transition length, we may plan the transition lengths of the curves to be increased. In such circumstances, change in length of transition will affect the versines of the circular portion of the curve also. Increased length of transition will also require increased shift, which varies with square of the length of transition. If we wish to accomplish the task with least amount of efforts, this **problem of increasing the length of transitions becomes a special case of realignment of curves.**

**Note:** 1. Sometimes, from speed considerations, we need to reduce the degree of the curve. Such a case requires the entire curve to be set again and is not in the scope of realignment.

2. The realignment is almost entirely a problem in open line, to be executed in block periods. However, the construction work such as gauge conversion, upgradation of lines etc may also require realignment. During construction work, the aim shall be to have as perfect geometry as possible. Therefore, normal constraints of open line such as block availability, resources etc shall not dictate the solutions in case the construction work is being done even though this book discusses the realignment problem taking these constraints in mind.

**5.3 Objectives of curve realignment:** Between any set of tangent tracks, infinite number of curves are possible, as we can see in figure 5.1. As we increase the degree of curve, the curve keeps on getting shifted towards the apex and vice versa. Therefore, we have large number of solutions to any realignment problem from which we have to choose the most suitable solution. When carrying out realignment, we aim to choose the curve amongst the various options which requires the least amount of efforts while meeting the desired objectives.

The curve realignment is done keeping the following in mind:
The realignment of curve aims at improving the geometry so that the variations in versines is gradual and maximum value of versine variation is within limits prescribed for the curve. The original designed versines in the curve are a good guidance when deciding the correction in versines, but the same cannot be the main criteria and other aspects have to be seen in deciding the proposed versines. In some cases, trying to restore the original geometry may be very tedious and costly, not worth the efforts, or it may not be feasible due to subsequent developments such as insertion of points and crossings or OHE masts, signals or other installations. Therefore, the aim of realignment of curves is not to restore the curve to original geometry, but to some desirable geometry.

5.4 Defects in Versines: If the versines in a curve are bad, the same can be identified by—

a) Unsatisfactory running on the curves by engine foot plate/ last vehicle inspection.

b) Unsatisfactory readings as per TRC/OMS/ Oscillograph runs.
c) Visible observation during routine push trolley/foot inspection.

d) Measurement of versines of a curve.

In a curve, the originally designed values of the versines for a curve are not as important as the station-to-station versine difference.

The service limits for the station to station versine variations laid down in IRPWM are:\(^1\)

<table>
<thead>
<tr>
<th>Speed Range</th>
<th>Limits of station to station variation (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Below 140 kmph and up to 110 kmph</td>
<td>10mm or 20% of the average versine on circular curve \textit{whichever is more}</td>
</tr>
<tr>
<td>Below 100 kmph and up to 50 kmph</td>
<td>15mm or 20% of the average versine on circular curve \textit{whichever is more}</td>
</tr>
<tr>
<td>Below 50 kmph</td>
<td>40mm or 20% of the average versine on circular curve \textit{whichever is more}</td>
</tr>
</tbody>
</table>

In case the exceedence of the above limit is observed during an inspection, local adjustments may be resorted to in case where the variation of versines between adjacent stations is only at few isolated locations, at the earliest possible. If more than 20\% of the stations are having versine variation above the limits prescribed, complete realignment of the curve should be planned \textbf{within a month}.

\textbf{Note for Maintenance:} The limits of versine variation given above are not for safety and this is demonstrated by the fact that one month is given even after the values are exceeded at more than 20 \% of the stations.

\textbf{5.4.1 Record of Curve Survey:} The curves shall be measured as per schedule laid down in IRPWM Para 107(4), 124(4), 124 A(4), 139(3) and 139 A(4), or whenever the curve appears to be unsatisfactory during inspections. The record of curve survey for realignment shall be kept in a curve register. The record should be collected in the following form:\(^2\)

\begin{itemize}
\item Curve from km..................to km..................
\item Between station...............and station............
\item Date of survey................
\end{itemize}

\(^1\) IRPWM para 421
<table>
<thead>
<tr>
<th>Station at Half-chord intervals</th>
<th>Versine in (mm)</th>
<th>Cant existing</th>
<th>Remarks regarding restrictions to slewing</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>-3</td>
<td>Zero</td>
<td>Points and Crossing at station no -2</td>
</tr>
<tr>
<td>-1</td>
<td>+2</td>
<td>-2 mm</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>Zero</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>5mm</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>10mm</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>20mm</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>10</td>
<td>25mm</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>11</td>
<td>29 mm</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>23</td>
<td>35 mm</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>36</td>
<td>40 mm</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>28</td>
<td>40 mm</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>35</td>
<td>38 mm</td>
<td></td>
</tr>
<tr>
<td>:</td>
<td>:</td>
<td>:</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>23</td>
<td>41 mm</td>
<td>Girder Bridge, obligatory point. Maximum slew ±50 mm</td>
</tr>
<tr>
<td>21</td>
<td>31</td>
<td>43 mm</td>
<td></td>
</tr>
<tr>
<td>22</td>
<td>33</td>
<td>45 mm</td>
<td></td>
</tr>
<tr>
<td>:</td>
<td>:</td>
<td>:</td>
<td></td>
</tr>
<tr>
<td>40</td>
<td>38</td>
<td>38 mm</td>
<td>High Bank, 4 m height, between station nos 40 to 60</td>
</tr>
</tbody>
</table>

The above record shall mention all important factors which might be relevant to the realignment including location of track features such as points and crossing, girder bridge, level crossing etc, high bank, deep cutting, structures nearby such as platforms, road over bridge, building, etc. If, in some case, it is felt that tangent track in the vicinity of the curve is not having proper alignment, and the curve has got extended into the tangent track, a few stations beyond the curve shall be included in versine readings taken for the curve so that these are rectified along with the realignment of curve. **In any case,**

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2 Para 422, IRPWM
whenever we are measuring the curve, the same shall be measured up to 2/3 stations ahead of and 2/3 stations beyond the curve so as to ascertain that the tangent track is actually correct and the curve has not extended into the tangent track.

5.5 Rectification of Curved Alignment: When rectification of alignment is required to be done in the straight alignment, the same can be done using theodolite or fishing cord. We can take any two points on good alignment and proceed to set out the alignment of all points in between. PWIs are even able to carry out a reasonable job of rectification of the alignment in straight by eye-sight only.

However, when the rectification of the alignment in case of curves comes, the human mind is not able to comprehend the alignment by eyesight and no attempt shall be made to rectify the track alignment in curve by eyesight. The rectification shall be done only by proper measurements using theodolite, fishing cord etc. Further, as discussed above, the effect of the rectification of track alignment at a point on the adjoining stations must be considered and hence proper computations must be done before taking up the rectification of curved alignment.

Before we proceed further, a few terms associated with the realignment need to be defined:

a. Slewing: Shifting the alignment of track is also called slewing. The slewing can be in either direction, outside of the curve or inside of the curve. When the track is slewed outwards, away from the center of curve, it leads to sharpening of the curve as the radius reduces and curvature increases. On the other hand, the shifting of the track inwards, towards the center of curve, results in easing of the curve as the radius increases and curvature reduces.

b. Sign convention for slews: In this book, as a standard sign convention, the inward slew is considered as positive slew whereas outward slew is considered as negative slew.

c. String lining operation: The method of determining the corrections to be made in the alignment of a curve, after measuring the versines and the rectification of the alignment thereafter is called string lining operation.
Note: The procedure given in para 5.6 and 5.7 below is the basic mathematical treatment to the subject of realignment. The computer programs use these for computations. Knowledge of the same will enable the engineer to be able to better understand and control the output of computer programs. Since these days computers are widely available, it is not expected that the curve realignment problem will be solved by hand even in field. Therefore it is requested that the field engineers go through the para 5.6 for their information and seek to understand the basics rather than learn the procedure by heart.

5.6 String Lining for Rectification of curved alignment: The string lining method has the following stages:

- Measurement of versines of existing curve.
- Consideration of Obligatory points.
- Determination of the revised alignment and computation of slews.
- Slewng of the curve in field to the revised alignment. (The detailed procedure for this is explained in para 2.9, Chapter II)

5.6.1 Measurement of existing versines for string lining: Before starting the measurement of versines, the gauge correction shall be carried out. After gauging is done, the versines are measured. As the geometry tends to get disturbed in the tangent track adjoining the curve, a few stations ahead of curve and behind the curve on the tangent track shall also be included in the versine readings. These readings will also help when the length of curve may have to be increased on either side for proper rectification of the curve geometry.

5.6.2 Consideration of Obligatory Points: While measuring the versines, the features that restrict the extent to which the track can be slewed in either direction shall also be noted down. Such points are called obligatory points. The locations where such restrictions are there include girder bridges, point and crossings, adjoining track, permanent structure adjoining the track or limitation of railway land boundary, OHE mast, signal post, platform, FOB/ROB column/abutment etc. The obligatory points may not permit any slew or only restricted slew. Many obligatory points have restriction
in slew only on one side. The maximum amount of slew is normally restricted in the OHE area to \( \pm 50 \text{mm} \) without alteration in OHE.

**Note:** It must, however, be remembered here that any obligatory point is obligatory only in respect of costs and benefits. This means that there is a cost involved in removing any obligatory point. The benefit of improvement in geometry, and thereby the speed/comfort of movement, is to be weighed against the cost required to remove the restriction before taking any decision regarding any track feature before declaring the same as obligatory point. If the shifting of curve gives benefit such as removal of a permanent speed restriction etc, it will be worthwhile to put in efforts and shift the feature which is restricting the slewing of the curve.

### 5.6.3 Working out the revised alignment and computation of slews:

Based on the measurement of existing versines, the proposed versines have to be chosen. To choose the revised alignment, the exercise of computing the first and second summations of the versine readings is to be done.

**(a) First Summation of Versines:** The first summation of versines at a station is the sum of all versines upto the station. In this chapter, the same is represented as FSV with subscript ‘e’ for existing and ‘p’ for proposed versines.

<table>
<thead>
<tr>
<th>Table 5.1: Computation of first summations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Station No</td>
</tr>
<tr>
<td>0</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>.</td>
</tr>
<tr>
<td>.</td>
</tr>
<tr>
<td>.</td>
</tr>
<tr>
<td>n</td>
</tr>
</tbody>
</table>
The first summation computation procedure is shown in table 5.1 above.

(a) Physical Meaning of First Summation of Versines:

As seen in table 5.1, FSV upto a station is the sum of all versines upto that station i.e. for station 'n', FSV=V₀ + V₁ + V₂ + ... + Vₙ₋₂ + Vₙ₋₁ + Vₙ

If we look at the figure 5.2, it is clear that if we divide the versine diagram into histograms with each segment equal to the distance between the stations, the area of the versine diagram is:

Distance between stations x (V₀ + V₁ + V₂ + ... + Vₙ₋₂ + Vₙ₋₁ + Vₙ).

i.e. First summation of versines upto a station represents the area of versine diagram upto the station (in station distance units).

(b) Second Summation of Versines: The second summation of versines at a station is the sum of all first summation of versines upto the previous station. In this chapter, it is represented as SSV with subscript 'e' for existing and 'p' for proposed versines.

If station to station distance is taken as unit,
Area of each histogram segment = Ordinate at center i.e., V₀, V₁, V₂, ..., Vₙ etc
Total Area of the versine diagram = Sum of areas of each histogram segment
= V₀ + V₁ + V₂ + ... + Vₙ₋₂ + Vₙ

Lever Arm for each histogram segment = n - station number
Moment of versine diagram about station n = n* V₀ + (n -1) * V₁ + (n -2) * V₂ + .... + 2* Vₙ₋₂ + Vₙ₋₁

Fig. 5.2
Table 5.2: Computation of first and second summations

<table>
<thead>
<tr>
<th>Station No</th>
<th>Versine</th>
<th>FSV</th>
<th>SSV</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>V₀</td>
<td>V₀</td>
<td>-</td>
</tr>
<tr>
<td>1</td>
<td>V₁</td>
<td>V₀+V</td>
<td>V₀</td>
</tr>
<tr>
<td>2</td>
<td>V₂</td>
<td>V₀+V₁+V₂</td>
<td>2*V₀+V₁</td>
</tr>
<tr>
<td>3</td>
<td>V₃</td>
<td>3<em>V₀+2</em>V₁+V₂</td>
<td></td>
</tr>
<tr>
<td>.</td>
<td>.</td>
<td>.</td>
<td>.</td>
</tr>
<tr>
<td>.</td>
<td>V₀+V₁+…+Vₙ₋₁</td>
<td>Vₙ₋₁</td>
<td></td>
</tr>
<tr>
<td>n</td>
<td>Vₙ</td>
<td>V₀+V₁+…+Vₙ</td>
<td>n*V₀+(n₋₁)*V₁+(n₋₂)<em>V₂+…+2</em>Vₙ₋₂+Vₙ₋₁</td>
</tr>
</tbody>
</table>

The second summation computation procedure is shown in example 5.2 above

**Physical Meaning of Second Summation of Versines:** As seen in table 5.2, SSV upto a station is the sum of all versines upto previous station. For station ‘n’, SSV= n* V₀+(n₋₁)*V₁+(n₋₂)V₂+…..+2*Vₙ₋₂+Vₙ₋₁. If we see the figure 5.2 again, the lever arm for V₀ upto the last station is n. The lever arm for V₁ is (n₋₁) and so on and the lever arm for Vₙ₋₁ is 1. This means that the expression for second summation is also the moment of the area of histogram derived from the versine diagram about the last station. i.e. SSV upto a station represents the moment of versine diagram upto the station (in station distance units).

**5.6.4 Properties of Versines of a curve:** The proposed versines are to be chosen keeping the following basic properties of versines of curved track in mind:

**a) First Property of Versines of any Curve:** Slewing a station on curve affects the versines at the adjoining two stations by half the amount in opposite direction. Let us examine what happens when we slew a point on the curve. Let us consider a curve with stations A, B, C, D and E at any point on the curve each at one station unit distance between them. If we slew the station C to C’ without disturbing any other point on the curve, the versines are affected at station B, C and D, as shown in figure 5.3.
Let the point C be slewed outwards by an amount $CC'$. The chord $AC$ gets shifted to $AC'$. The change in the versine at B is $bb'$.

Consider $\triangle ACC'$, since the radius of railway curves is very large, we can consider $CC'$ as parallel to $bb'$ (the figure 5.3 is not drawn to scale and the lines do not appear to be parallel but in field, the assumption is quite valid).

$\therefore$ The triangles $ACC'$ and $Abb'$ are similar, i.e. $\frac{bb'}{CC'} = \frac{AB}{AC}$

Now, since the radius of the railway curves is quite large as compared to the distance between the stations, the chord $Ab \sim AB$ and if $c$ is the chord length, $AB = c/2$ and $AC = c$

$\therefore \frac{bb'}{CC'} = \frac{1}{2}$ i.e. the slew at station B is half that at station C.

![Fig. 5.3](image)

Also, looking at figure 5.3, it is clear that when the station C is slewed outwards (versine at station C increases), chord AC shifts towards the station B and vice versa.

Similarly, the change in versine at station E also changes by an amount equal to half the amount at D, and the change will be in opposite direction to the change at station C.
Therefore it is clear that whenever we slew the curve at a station, the versines at the stations at either side get affected by half the value of slew in opposite direction. Whenever we disturb the curve, we have to keep this property in mind. For example, if we slew curve at station n by 10mm outwards i.e. versine increases by 10mm at station n, the versines at station (n-1) and (n+1) reduces by 5mm each.

b) Second Property of Versines of any Curve: In curves between same set of tangents, for equal chord lengths the sum of versines is constant. The same can be proved as follows: Let us consider a curve with only 3 stations as shown in figure 5.4.

If chord length is c, the versines can be worked out as -

\[ V_0 = \frac{c}{2} \tan \alpha \equiv \frac{c}{2} \alpha \]
\[ V_1 = \frac{c}{2} \tan \beta \equiv \frac{c}{2} \beta \]
\[ V_2 = \frac{c}{2} \tan \gamma \equiv \frac{c}{2} \gamma \]

\[ \therefore \Sigma V = V_0 + V_1 + V_2 = \frac{c}{2} \alpha + \frac{c}{2} \beta + \frac{c}{2} \gamma \]

Now, in a triangle, it can be demonstrated that the external angle is equal to the sum of the opposite two angles. \( \therefore \) If we
consider $\triangle$ IHJ, $\angle$ RIJ = $\angle$ IHJ + $\angle$ IJH = $\alpha$ + $\alpha$ = 2 $\alpha$

Similarly, in $\triangle$ JKL, $\angle$ RKJ = 2$\gamma$ and in $\triangle$ IRK,$\angle$ RIK+$\angle$RKl
i.e. $\Delta$=(2$\alpha$+$\beta$)+($\beta$+2$\gamma$)

» $\Delta$ = 2[$\alpha$+$\beta$+2$\gamma$]

» $\Delta$/2 = [$\alpha$+$\beta$+2$\gamma$]

Substituting in the equation above,

.: $\Sigma V$ = (c/2)*($\Delta$/2) = $\frac{c \times \Delta}{4}$

This means that the sum of versines for a curve depends only on the chord length and the deflection angle (i.e. a curve between the same set of tangents), This proves the second property of the realignment of curves for a curve with three stations.

From the first property of the versines of any curve, it comes out that the sum of the versines in any curve will remain constant even when it is disturbed at any one point. Starting from a three station curve shown in figure 5.4, we can progressively slew the curve, one station at a time and the sum of versines will always remain constant, regardless of the actual length/ no of stations. This property means that whatever be the actual shape of the curve and/or the location of the starting/ end point of the curve, the summation of the versines remains constant.

As a corollary to the above property, whenever we choose any solution to realignment problem and propose new versines, the sum of the proposed versines has to be kept same as that of existing versines. This corollary is used when we choose a solution to the realignment problem.

c) Third property of Versines of any Curve: Twice the second summation of the difference of proposed and existing versines upto a point gives the slew at a station: To demonstrate this property, let us draw the curve from station no -1 to station no 3 and only one tangent in figure 5.5.
The figure 5.5 is showing deflection of the curve from the straight (tangent). The curve is having stations marked -1, 0, 1, 2 and 3. AB is the tangent track and BK is the extension of the same towards the apex. Points E, I and N correspond to stations 1, 2 and 3 on the curve. AE is the chord stretched to measure the versine at station 0. Similarly, BI and EN are chords stretched to measure versines at station no 2 and 3 respectively. Line, BEHL, joining stations 0 and 1 and extended further is drawn. Another line EIM, joining stations 1 and 2 and extended further is drawn.
Let the versine at station 0, i.e. BC be \( V_0 \).

By geometry, consider similar triangles ABC and ADE.

Since \( AD = 2 \) station units and \( AB = 1 \) station unit, \( AD = 2 \times AB \)

\[ \therefore \ DE = 2xV_0. \]

Further consider similar triangles BDE and BGH.

Since \( BD = 1 \) station unit, and \( BG = 2 \) station units, \( BG = 2 \times BD \),

\[ \therefore \ GH = 2xDE. \text{ i.e. } GH = 2 \times x2V_0 \text{ and } \]

In similar triangles BDE and BKL,

since \( BD = 1 \) station unit, and \( KL = 3 \) station units, \( BK = 3 \times BD \),

\[ \therefore \ KL = 3xDE. \text{ i.e. } KL = 3 \times 2xV_0. \]

In the similar manner, let the versine at station 1, i.e. EF be \( V_1 \).

By geometry, consider similar triangles BEF and BHI.

Since \( BH = 2 \) station units and \( BE = 1 \) station unit, \( BH = 2 \times BE \)

\[ \therefore \ HI = 2xV_1. \]

Further consider similar triangles EHI and ELM.

Since \( EH = 1 \) station unit, and \( EL = 2 \) station units, \( EL = 2 \times EH \),

\[ \therefore \ LM = 2xHI. \text{ i.e. } LM = 2 \times 2xV_1. \]

And, let the versine at station 2, i.e. IJ be \( V_2 \).

By geometry, consider similar triangles EIJ

Since \( EM = 2 \) station units and \( EI = 1 \) station

\[ \therefore \ MN = 2xV_2. \]

Now let us consider the deflection (offset) of the curve form the straight (tangent):

Offset at station 0: 0

Offset at station 1: \( DE = 2xV_0 \).

Offset at station 2: \( G-H-I \sim GH+HI = 2x2xV_0 + 2xV_1 \) (There is a small error here but since the railway curves have very large radii, the error is small and can be neglected).

Offset at station 3: \( K-L-M-N \sim KL+LM+MN = 3x2xV_0 + 2x2xV_1 + 2xV_2 \).

Continuing the trend further,
Offset at station n: \( nx_2V_o + (n-1)x_2V_1 + (n-2)x_2V_2 + \ldots + 2x_2V_{n-2} + 2xV_{n-1} \)

\[ \Rightarrow \text{Offset at station n: } 2x[nV_o + (n-1)V_1 + (n-2)V_2 + \ldots + 2V_{n-2} + V_{n-1}] \]

Now, since the equation in the bracket, i.e. \( nV_o + (n-1)V_1 + (n-2)V_2 + \ldots + 2V_{n-2} + V_{n-1} \) is the second summation of the versines of a curve, it follows that:

Offset at station n from the straight: \( 2 \times \text{Second summation of versines up to station n} \)

This means that twice the second summation of versines of a curve up to a station represents the offset of curve from tangent at that station.

If we look at the fig 5.6, the solid curve represents curve as measured in field. Let the dotted line represent the desired geometry of the curve. Now the offset of the existing (solid) curve form straight is given by twice the second summation of the existing versines. Whereas the offset of the proposed (dotted) curve from straight is given by twice the second summation of the proposed versines. This means that if we calculate twice the difference of second summations of the existing and proposed versines, we get the slew at a station. This is the third property of the realignment of versines of any curve.
d) **Fourth property of Versines of any Curve:** The second summation of the difference of proposed and existing versines at first and last stations shall be zero: This property is not sacrosanct but is meant to isolate the curved track from straight alignment. This ensures that while carrying out the realignment, we have isolated the curved track and are trying to solve the alignment problem of this portion of the track alone. If the slew is not zero at the first and last stations, it would imply that we are attempting to realign the straight portion also. In such a case, the problem in the curved track will be transferred to the straight portion of the track, and the problem will not be possible to be solved unless the complete straight is aligned with this changed geometry.

**Note:** Here the first and last stations do not mean the start and end of the curve but the first and last station included in the curve survey. It is for this reason that it is mentioned above that the disturbed components of tangent track (straight) adjoining the curve which are lying outside the original curve length shall also be included in the curve survey.

If it is found that the tangent track (straight) is not in correct alignment, the same may be rectified using theodolite or fishing chord or by sight and the problem of realignment of curve shall be isolated. And therefore, we have to ensure during solving the realignment problem that **the second summation of the versines at the first and the last stations shall be zero.**

5.6.5 Proposed Versines by the String Lining Operation:
Based on the above properties of versines of any curve, the proposed versines shall be designed. The versines measured in field is termed as existing versines and the desired versines are called proposed versines. The steps in choosing proposed versines are as follows:

**a)** The versines over the disturbed curve are tabulated and summed up.

**b)** As per the second property, the proposed versines shall be chosen close to the average of versines over the circular portion of the disturbed curve. The versines on transitions are then designed with a uniform rate of variation over the stations on the proposed transitions commencing at or near the apparent
tangent points at both ends. The versine distribution shall be as shown in figure 5.7.

Fig. 5.7 : Existing and proposed versine diagram

Note: The shape of the proposed versine diagram as above shall be as close as possible to the theoretical trapezoidal distribution (as shown in figure 1.15, Chapter I).

c) If there is any small difference between the sums of the existing and proposed versines, due to rounding off errors, the balance of versines after deducting the total of proposed versines from the total of existing versines shall be distributed symmetrically about the centerline of the curve.

d) Difference between the existing and proposed versines shall be found for all the stations.

e) The first summation of the difference of the proposed versines and existing versines shall be found out. If the calculations are done correctly, the first summation shall be zero at the last station, in accordance with the first principle of curve realignment.

f) Then the second summation of the difference of the proposed versines and existing versines shall then be found out.

If the proposed versines are suitable, the second summation at last station shall be zero in accordance with the fourth property of the versines of a curve. However, since the
curve as measured is disturbed and the solution that we have assumed is ideal trapezoidal distribution of versines, there are chances that the solution assumed is not the suitable one. Quite often, a residual slew either outward or inwards at the last station of the curve is left. This residual slew is required to be eliminated by using what is known as “correcting couple”. A correcting couple is a set of two equal and opposite corrections to versines applied at two stations such that the residual of second summation at the last station is reduced to zero.

5.6.6 Physical meaning of correcting couple : When there is residual second summation of versine difference at the last station, it means that that the second summations of the existing and proposed versines are not equal, i.e. the center of gravity of the exiting versine diagram does not coincide with the proposed versines. The correcting couple [which is a small correction in the proposed versine diagram (figure 5.8)] means a small transfer of the area of versine diagram such that the centers of gravity of the existing and altered proposed versine diagrams match.

![Fig. 5.8](image)

Another way of looking at the things is if we look at the curve in plan as shown in figure 5.9. When the proposed versines 1-1’ is not the correct solution to the realignment problem, the second summation of the proposed curve at the last station (proposed offset from tangent) is not coinciding with the
second summation of the existing curve (existing offset from tangent) and the point 1’ is not on the second tangent. The correcting couple changes the second summation at all the stations subsequent to the first station of application, i.e. the offset of the curve changes at all these points and the effort is such that the end of the curve at 2’ falls on the second tangent. In this analogy, the correcting couple means that the proposed curve is held at a point and turned about this point in such a manner that the degree of the curve remains the same at the other points but the end of the proposed curve meets the existing curve.

Fig 5.9

A few questions regarding how to choose correcting couples must have arisen in the minds of the readers, which are answered below:

a) Why correcting couple?: The correction in proposed versines can also be accomplished by changing the proposed versines and doing the complete calculations again. However, we are not sure as to what might be the correct solution for the proposed versines and we might have to do a number of iterations. For ease of calculations, we separate out the calculations for the corrections in the form of correcting couple.

b) Why correcting couple has to be equal and opposite?: The sum of the versines must remain constant as per the first property of versines of a curve, hence whatever correction is applied, must have sum zero. Therefore, the correcting couple has to be equal and with opposite signs.

c) What shall be sign of the correction applied first?: In the tabulated versines, the two elements of correcting couple are applied one above the other. As discussed above, the two have to be of opposite signs. The correction applied above the
other must have the sign opposite to the residual second summation value at the last station so that the second summation of the correcting couple is having sign opposite to the residual second summation of proposed versines.

d) Where shall correcting couple be applied?: The capacity of a correcting couple to reduce the residual slew at the last station depends on the distance between the two corrections. Therefore, the correction shall be applied as far away from each other as possible.

The correcting couple is to be chosen in such a manner that the residual second summation of the versine difference and the correcting couple at the last station gets reduced to zero.

5.6.7 Principles for choosing correcting couples: To summarise, the correcting couple shall be chosen such that

a) These are equal and opposite.

b) If the residual second summation of versine difference at the last station is positive, a negative correction is applied at the initial stations of the curve and a positive correction is applied at the latter stations of the curve, such that the second summation of the correcting couple is negative. For a negative residual value of second summation of versine difference, the corrections with opposite signs shall be applied.

c) The correcting couple disturbs the trapezoidal distribution of the proposed versines, and consequently the correction shall be as small as possible in value. For this purpose, the two corrections are applied at stations which are as far apart as possible. If the corrections are applied farther apart, the second summation of the correcting couple will be higher.

d) If it is found that the one correcting couple is not sufficient to counteract the residual second summation of the versine difference at the last station, apply more correcting couples. The further couples are also to be applied in as small absolute value as possible and with as much distance between them as possible.

e) Above is continued till the second summation of the
correcting couple becomes equal to the residual value of the second summation of versine difference at the last station. Now the second summation of the versine difference and second summation of the correcting couple are summed up. This gives the half slews at each of the stations in accordance with the third principle of realignment of curves. The slews at all stations are found by doubling the half slews so found out.

5.6.8 Limitation of choosing proposed versines manually:

a) The above procedure of choosing the proposed versines manually, without any mathematical assistance can be used conveniently only when the length of curve is small or if we have to tackle small portion of the curve. However when we take up longer curve lengths, choosing the proposed versines by above method becomes very difficult.

b) Further, in field, the curves often get too disturbed and shifting the beginning and end of curve may be required so as to have the proposed curve as close as possible to the shape of the existing curve, and we get minimum slews. This cannot be done if we use the method described above.

c) It requires tedious repeated iterations to design a correct proposed versine and to commence the curve at the appropriate point for the versine chosen such that:

- There is no residual slew at the other end, and
- The maximum slew, either outside or inside, does not exceed the practical limits, say 150mm.

To achieve these objectives, we must use a mathematical approach for designing the proposed versines. This is called optimization of the realignment solution.

5.6.9 Optimization of curve realignment solution:

If the realignment is attempted by the procedure outlined above, one of the following situations may arise:

(a) Improper choice of Beginning of Curve: When the proposed versine chosen happens to be correct but the BC chosen is not correct, it results in heavy residual slew at the end, as shown in figure 5.10.
Fig. 5.10: Effect of choosing different beginning of curve

In the figure above, existing curve shown in solid line and proposed curves shown in dotted.

**BC**: Beginning of Curve

**EC**: End of Curve

**CC**: Center of Curve

1 - 1 : Disturbed curve

2 - 2 : When BC chosen happens to lead the correct BC. The end of curve shifts inwards.

3 - 3 : When BC chosen happens to trail the correct BC. The end of curve shifts outwards.

4 - 4 : Proposed curve with correct BC and EC symmetrical about CC

*Note*: Curves 3-3 & 2-2 are not symmetrical about CC

(b) Improper Choice of versine: When the BC chosen is correct but the proposed versine chosen is not correct, it results in heavy slews outward or inward over the entire curve. The effect of different versine readings is shown in figure 5.11.
1-1: Disturbed curve

2-2: Proposed curve when versine chosen is less than required

3-3: Proposed curve when versine chosen is more than required

4-4: Proposed curve with correct versine

**Note:** All the three curves, 2-2, 3-3, 4-4 are symmetrical about (CC). The curves 2-2 and 3-3 are away from the existing curve and hence the slews are more.

**(c) Choosing Correct Beginning of Curve and Proposed Versines:** As discussed above, success in obtaining the most suitable revised and rectified alignment in string lining operations depends not only on the correct choice of versines but also on the correct choice of the starting point.

To enable decision making regarding the correct BC and versines, concept of fixing a point on the curve, usually center of curve (CC) is used. The following procedure is adopted for the same:

**Step I: Find the chainage of the centre of the existing curve (CC):** The center of curve used for the realignment solution is not the physical center of the length of curve. The **CC is the x-coordinate of the center of gravity of the versine diagram.**

The center of gravity of existing versine diagram can be determined by dividing the moment of the versine diagram about the last station by the area of the versine diagram. In order to get the value of CC in reference to the first station, the center of gravity thus found has to be deducted from n(last station of the existing curve).

\[
CC = n - \frac{\text{Second Summation of } V_e(\text{SSV}_e) \text{ upto last station(n)}}{\text{First Summation of } V_e(\text{FSV}_e) \text{ upto last station(n)}}
\]

......(5.1)

**NOTE:** As assumed, for making the correct choice of BC and proposed versine, the CC is not disturbed. Therefore, the CC found out in step I is also the CC for the proposed versine diagram.
Step II: Find the offset from tangent at CC for existing curve: As per the third principle of curve realignment, the offset of a station from the tangent is given by the second summation of the versines up to that station. Using this principle, the offset of the CC for existing curve can be found out from the second summation worked out for the existing versines. If CC happens to fall between two stations, the offset at CC shall be found by interpolation of the values at the adjoining two stations.

Step III: Find the offset at CC for the proposed curve: As we desire to keep the center of curve undisturbed, the offset at CC for the proposed curve shall be equated with the offset at CC for the existing curve. (Physically, this means that we aim at maintaining constant center of gravity of the curve. It may be recalled that this is what we aimed at when the correcting couple was being applied in para 5.6.1(f). Here the same is being done before the versines are chosen). The offset at CC for the existing curve has already been found in step II and the offset for the new curve can be found out if we consider equivalent circular curve. Equivalent circular curve is the proposed circular curve if there were no transitions. This curve has been used in para 1.11, chapter I when we discussed the concept of shift. The equivalent circular curve is shown as dotted line in figure 5.12. The equivalent circular curve is used as it is easier to do computations for the same.

Fig. 5.12: Equivalent circular curve

Let the length of the proposed equivalent circular curve be N station units. Let the uniform versine in circular
curve be V and L be the length of transition curve in station units.

If T is the tangent length upto CC, for the equivalent circular curve, the offset at center of curve: \( O_c = \frac{T^2}{2R} \)

Since the actual proposed curve is also having transitions at either end, the equivalent curve shifts inwards when the transition is introduced by an amount equal to shift, S.

Using equation (1.21), Offset at CC for the proposed curve =
\[ O_c + S = \frac{T^2}{2R} + \frac{L^2}{24R} \]

The minimum length of transition for any curve is known from the speed potential of the section, and the designed cant actual/ cant deficiency values using formulae in the chapter I/II.

The value of L chosen shall be
- more than the minimum required from the speed considerations
- based on the trend of the existing curve versine readings.

**Step IV: Equate the offset at CC for the existing and proposed curves:**

From equation (1.2), versine \( V = \frac{C^2}{8R} \)

Considering \( C = 2 \) station units, \( V = \frac{2^2}{8R} = \frac{1}{2R} \) .................(5.3)

For the railway curves, the radius of curve is very large, so the tangent length, T can be approximately considered to be equal to half the curve length without appreciable error

i.e. \( T \approx \frac{N}{2} \)

\[ \therefore \text{The equation (5.2) reduces to Offset at CC} = \]
The versines are designed by slope method and it is assumed that the versine starts from zero to the slope of the versines in the transition portion. However, in actual practice, the vehicles have bogies and these will moderate the effect of any sudden introduction of the versines and the actual bogie traveling over the transition experiences slightly different versines. This effect is the same as discussed in para 1.11.1, chapter I while discussing the phenomenon of virtual transition. Therefore, the equation (5.4) gets slightly altered and the final value comes to

\[
O_c + S = \frac{VN^2}{4} + \frac{V(L^2 - 4)}{12}
\]

i.e. Offset at CC for proposed curve

\[
= \frac{V^2N^2}{4V} + \frac{V(L^2 - 4)}{12} \]

The total versine for the equivalent circular curve is \( V \times N \),

From the first principle of curve realignment, the sum of versines for any curve between the same set of tangents for equal chords is equal, therefore, sum of versines of the existing curve.

Therefore, \( V \times N = \text{FSVe} \)  

\[ \text{Offset at CC for existing curve} = \frac{(\text{FSVe})^2}{4V} + \frac{V(L^2 - 4)}{12} \]  

Since, offsets for the existing and proposed curve are to be equal, the LHS of the equation is known from step II above,

\[ \text{Offset at CC for proposed curve} = \frac{(\text{FSVe})^2}{4V} + \frac{V(L^2 - 4)}{12} \]  

\[ \text{(5.7)} \]
We have already found out the offset at CC for the existing curve in step II above, there is only one variable i.e. V in the above equation. The quadratic equation (5.7) can be solved to get the proposed versine in the circular portion of the curve.

**Step V: Find out the correct length of curve:** In step IV, the value of the proposed versine, V, in the circular curve is found out. Now, the length of equivalent circular curve can be found out using equation (5.6):

\[ \text{FSVe} = N \times V, \text{ i.e. } N = \frac{\text{FSVe}}{V} \]  

(5.8)

As shown in figure 5.11, the length of the proposed curve is found out by adding the designed transition (L) in station units to the circular curve length \( N' = N + L \).

**Step VI: Find out beginning and end of curve:** Now, Chainage of the beginning of the proposed transitioned curve (BC) is then found out as \( \text{(BC)} = \text{(CC)} - \frac{N + L}{2} \)  

(5.9)

And, Chainage of the end of the proposed transitioned curve (EC) is found out as \( \text{(EC)} = \text{(CC)} + \sqrt{\frac{N + L}{2}} \)  

(5.10)

**Step VII: Design of proposed versines:** The proposed versine diagram shall be as close to the trapezoid as possible. The versines in the circular portion of the curve shall be V and for the transitions, the proposed versines are found out using the versine slope \( \frac{V}{L} \). The versines are proposed for each station by rounding off the values obtained by the versine slope. In Case BC and EC do not fall on any station, the versines at the first/last stations can be found out by interpolation. Versines at other stations on the transitions can be designed by successive addition of value of versine slope, till the value reaches just short of designed versine for circular curve. The final versine diagram of the proposed versine diagram is shown in figure 5.13.
Since the versines are to be proposed in whole numbers or at the most to first decimal point only, there will be some rounding off involved. There might be some small error in the sum of the versines so designed versus sum of versines for the existing curve. Adjustment of this minor difference may be done symmetrically at appropriate number of stations either near about the center of curve or at the junction of circular and transition curves, depending on the total error. This adjustment shall be as small as possible.

Now that we have designed the proposed versines, further computations are done as per the procedure given in para 5.6.5.

The complete procedure enumerated above will be more clear from the example 5.1.

5.6.10 Passing the Curve through a desired point anywhere on the circular portion other than Center of Curve (CC):

The curve realignment problem cannot be solved without considering the site constraints. As discussed in para 5.6.2 above, it might not be possible to shift the curve at certain points called obligatory points. Let us take a curve as shown in figure 5.14 below, where X is the station on curve at which no slewing or shifting is possible.
For ensuring the site constraint, slew at station X shall be zero. i.e. Offset at station X for the existing curve = Offset at station X for the proposed curve.

As per the third principle of curve realignment, the offset of a station from the tangent is given by the second summation of the versines upto that station. Therefore, offset at station X for the existing curve can be found out from the second summation of the existing versines upto station X.

Using the same analogy as in 5.6.9 (c) and eq (5.5), offset at station X for the proposed curve = $O_x + S = \frac{T_x^2}{2R} + \frac{(L^2 - 4)}{24R}$, where $T_x$ is the length of the tangent upto X.

And since $V = \frac{1}{2R}$, offset at station X for the proposed curve = $VT_x^2 + \frac{V(L^2 - 4)}{12}$

In this case, $T_x = \frac{N}{2} + (X-CC) = \frac{FSVe}{2V} + (X-CC)$

∴ Second summation of the existing versines upto station X =
\[ V \left( \frac{FSVe}{2V} + (X - CC) \right)^2 + \frac{V(L^2 - 4)}{12} \quad \text{from equation} \ldots \quad (5.7) \]

This gives us a quadratic equation for proposed versine for passing the curve through station X of the existing curve. After having designed the proposed versine, BC, EC and versines over transitions can be found out as explained earlier in para 5.6.9(c). The procedure for the same is explained in the example 5.2.

5.7 Change in Transition length: The need for change in transition length is discussed in para 5.2. item 3. Increased length of transition means increased shift, which varies with square of the length of transition. Such shifting of the entire curve will require considerable energy especially if the curve length is more. Sometimes, such solution may not be feasible in open line and it will be desirable to have a solution which has minimum slews.

The additional shift of the curve on account of the increased transition length takes place away from the apex of the curve. If the degree of the circular curve is increased, the curve will be closer to the apex, as shown in figures 5.1 and 5.11. Therefore, the slews for this case can be reduced by making the curve sharper and shifting the entire curve in the opposite direction. In this solution, the higher slews over the increased transitions can not be avoided but due to additional shift caused by increase in transition length over the circular portion, slews are reduced.

In this case, the calculations shall be done as per the procedure outlined in the para 5.5. The increased transition length is considered in the procedure by choosing appropriate length of transition in station units. When we optimize the solution by keeping the slew at center of curve as zero, the circular portion of the curve gets sharper and the increased versine which is compensating the increased transition length is automatically obtained. The procedure is explained in example 5.4.

5.8 Local adjustment of curved alignment: When only a short stretch of the curve, say 100 to 150 meters long, gets disturbed it is neither necessary nor desirable to realign the
entire curve. It would suffice if the versines over the disturbed portion only are adjusted. As per IRPWM\(^3\), when the number of stations having station to station versine variation is less than 20% of the total number of stations on the curve, only local adjustments are to be carried out. If such an attention is not given in time, it will lead to disturbance of versines on the nearby stretches also in due course of time, necessitating realignment of the entire curve.

Further, if the running of a curve is bad and the resources for the complete realignment are not available, only part of the curve may be attended. To get the maximum benefit from the limited efforts put in a curve, tackling worst few locations as local adjustment provides immediate relief.

**Procedure for Local Adjustment:** The local adjustment can be done by solving a portion of the curve by keeping the basic principles of realignment enumerated above in mind. This is two stage procedure, similar to the realignment problem:

- Consider only the existing versines of the disturbed portion of the curve. While taking the disturbed portion, one or two stations on either side of the disturbed stretch where the versines are not disturbed much should be included.

- Find out the total of the existing versines for the disturbed curve.

- Divide the sum of existing disturbed versines by the number of stations on the disturbed curve and the same is the proposed versine at all the stations of the curve.

- Calculate the first summation and second summation of the versine difference as already described for the disturbed curve only.

- If there is any residual slew at the last station, eliminate the same by application of a suitable correcting couple as already explained.

The procedure is explained in example 5.3.

**5.9 Attention to transitions:** Due to the lateral curving and centrifugal/ centripetal forces, curves require more maintenance as compared to straight track. Within the curve,
transition portion requires maximum attention due to following reasons:

- There is a continuous change in the curvature/versines/cross levels and the lateral forces are varying constantly in the transitions.
- There is jerk at the entry and exit of the curves due to sudden introduction of the versines in cubic parabola.
- Parameters of versine and change in cross levels have certain permissible values from comfort/safety considerations. These values are same for the circular portion and the transition portion. In the transition portion of curves, there is already a designed change in curvature and cant. **This means that the margin for deterioration in the parameters on the transitions is correspondingly reduced and the transition portions are required to be attended more frequently as compared to the other portions of track.**

To attend to the transition portion of a curve alone, we shall take the end portion of curved track which is approximately 1.5 times transition length, starting from the apparent tangent point. Further procedure is similar to the local adjustment problem described in para 5.8 above, except that the proposed versines shall vary uniformly in the transition portion and shall be uniform in the circular portion. The typical curve in plan where the transitions are disturbed shall look like:

![Fig. 5.15: Disturbed transitions in a curve](image)

Since the length near transitions is disturbed, we take 1.5

---

³ Para 421(2), IRPWM
times length of transition for realignment, equal to O-X. The point O is the apparent tangent point (i.e. the point where the versines are almost zero). The curve upto point X can have some portion of circular curve also.

The proposed versines for the disturbed curve O-X shall be designed such that the slew at X is zero. This will ensure that the length of curve to be tackled is minimized and the transition length gets attended. Therefore, the SS of versine difference shall be zero at station X.

To simplify the computations, as earlier, let us consider an equivalent circular curve of length N having same sum of versines as the disturbed curve. Later on, we can add the length of transition. The transition curve length, L, is already known from the speed considerations.

Let us assume V is the versine for the equivalent circular curve. Since we are considering the curve upto station X only where the last station is having a large non zero versine value, only half of the last histogram of versine diagram shall be counted i.e. FS of the proposed curve upto station X shall be equal to

\[ \text{F.S. of Ve up to (X-1) stations} + \left( \frac{V}{2} \right) \]

Since the sum of versines for the existing and proposed curves shall be same, length of equivalent circular curve CL, will be given by: \( x \cdot V = \text{FS of } v_e \text{ up to stations (X-1)} + \frac{V}{2} \)

i.e. \( CL_x = \frac{\text{FS of } v_e \text{ up to stations (X-1)} + \frac{V}{2}}{V} \)

By equating offset at X for disturbed and proposed curves, we get: \( O_x = 2 \text{SS of } v_e \text{ up to } X = Vx^2 + \frac{V(L^2 - 4)}{12} \)
This becomes a quadratic equation for $V$ and can be solved. The beginning of curve can be found out as:

$$x - CL - \frac{L}{2} = x - \frac{F.S. of \ V_e \ \text{upto} \ (X-1) + \frac{V}{2} - \frac{L}{2}}{V}$$

Versines on transitions can now be designed as done earlier with minor adjustment for total versines up to stations $(X-1)$, if necessary.

The slews now can be worked out in the usual manner.

The procedure will be clearer from example 5.4.

The transitions require attention at closer intervals compared to the other parts of the track and this method can help the work to be done mathematically rather than by eyesight, which can lead to further disturbance of the curve. The regular attention to the transitions can help reduce the need for complete realignment at later date.

5.10 **Use of Computer Programs for Realignment of Curves:** The realignment process described in para 5.6 above is convenient to be done by hand only when the number of stations is less, say up to 40. Further, the hand calculations suffer from following limitations:

- Manual calculations are error prone
- The procedure is tedious and there is reluctance on part of engineer to do the iterations. Whenever we are carrying out the curve realignment, it is not always possible that we have the best solution in the first attempt. However, if we want to have a few alternatives and wish to compare these, the calculations have to be done again and again.
- Considerations of restrictions such as obligatory points or restriction on maximum slews makes the problem even more complex and very difficult to be solved by hand.

- As the number of stations increase, the calculations get very tedious.

To remedy the situation, the computers can and must be used. Computers provide freedom from donkey-labour part of the solution and the engineer can concentrate on the quality of output and refining the same. Using computers, solution to curve realignment problem is obtained in a matter of minutes and iterations can be done in a matter of seconds. Engineer can easily see various alternatives and compare them to choose the best suited solution.

The modern day computer programs have user friendly interface and their operation is quite easy with visual guides and help available for the same. In nut shell, computer programs make the realignment problem lot easier to tackle.

However, a note of caution: Use of computer programs does not eliminate the need for a permanent way engineer. The realignment solution cannot be found by just anybody. A good knowledge of the geometry, permanent way and the various site conditions/ restrictions is required to appreciate the various solutions that are obtained from the computer programs, and to choose the most convenient solution.

As already mentioned, computer programs mostly use the same procedures as enumerated above in para 5.6. The knowledge of this theory is not the pre-requisite for operating the computer programs but if the permanent way engineer is having the knowledge of the basics of how the computer program is working, he/she can better appreciate the results given by the computer programs and also, which parameter to alter so as to control the solution and get a more convenient and acceptable solution.

5.10.1 How Computer Programs work: As mentioned above, the computer programs basically use the same algorithm or basic logic as the detailed procedure given in the para 5.6 above while meeting all constraints or obligatory points on the
curve. Further, the computer programs do the optimization of the solution. The optimization of the realignment solution is generally done to:

(i) Reduce the maximum slew so that the realignment solution can be practically implemented.

(ii) Reduce the total slewing effort

A variety of computer programs have been developed by the Indian Railway engineers and are in use by the field engineers. The most popular amongst the available programs are:

(i) Program by Dr. M. Sheshagiri Rao, Retd Chairman, RITES (IRSE Officer of 1957 exam batch) in DOS environment


(iii) RC 100

(iv) Program in excel sheet by Shri Venkateswara Rao, (IRSE Officer of 1988 exam batch)

(v) Some tamping machines such as 3X machine and new DUOMATIC machines are having on-board computers and employ a software called WINALC. These machines take a measuring run and give solution for realignment after the same.

(vi) Other computer programs are available from the dedicated railway layout software such as the MXRAIL.

The different programs have different algorithms and the parameters used for optimization are different, e.g.:

(i) **Maximum value of slew (positive) shall be nearly equal to maximum value of slew (negative).** This optimization is used in the "true circular curve" option of the program by Shri M.S. Ekbote.

(ii) **Absolute sum of all slews shall be minimum:** This optimization is used by RC100.
(iii) Root mean square value of slews shall be minimum: This optimization is used by the program by Dr. M. Sheshagiri Rao.

(iv) Limit the maximum slewing effort at any one station: This optimization is used by the program developed in excel developed by Shri K. Venkateswara Rao. In his own words, the principle is “propose the versines which are same as existing versines at all stations except at two stations which cause the maximum versine variation and do usual double summation calculations. Repeat the process a number of times”. This algorithm has also been incorporated in one of the options in the realignment problem developed by Sh M S Ekbote.

(v) Averaging method: This is also not a holistic approach to solution of the curve but in some cases where the existing curve is badly distorted and there are severe site restrictions, this method will give some solution which is practical. In this method, even though the resulting geometry is not very good, it is much better than the smoothening mode tamping that is the only option left otherwise. In this method proposed versines are taken as average versine of 3 stations and successive iterations are done at user's choice. This algorithm has been incorporated in one of the options in the realignment problem developed by Sh M S Ekbote.

(vi) Divide the curve into segments: The compounding method in the realignment program developed by Sh M S Ekbote gives an option wherein the curve can be divided into segments, and the realignment solution is found out by converting the curve into a compound curve. The segments have to be chosen such that the curve is separated at points where the average versines of the curve change. Some amount of engineering judgment is involved in choosing the compounding points. With some trial and error, the optimum location for the compounding points can be found out and an acceptable solution can be obtained.
All the above optimization methods give different benefits to the permanent way engineer. The first method of optimization balances the maximum value of outward slew with the maximum value of inward slew. Since the absolute value of slew is very important constraint in open line working, the method gives good solutions.

The second method of optimization reduces the total slewing effort or the effort required to slew the curve, even though a few points may require higher slews. This method controls the sum total of slews.

On the other hand, the third method of optimization reduces the total slewing effort by controlling the root mean square of slews.

The fourth and fifth methods are sometimes the only feasible solutions in open line electrified territory, even though in these methods, the curve cannot be improved beyond a certain point.

The sixth method is a very powerful tool and the permanent way engineer carrying out the realignment can very accurately control the solution of the curve. Depending on the type of curve, and the extent of disturbance, any of the approaches can give better or acceptable solution to the permanent way engineer.

5.10.2 Which computer program to use?: As discussed above, lots of computer programs (and more sub-program options) are available, which have been extensively used by Indian Railways people over the years. Based on the discussions held with the authors of the programs and guest officers attending the various training programs at IRICEN, as also the field engineers who regularly use the computer programs for realignment, it has been concluded that the realignment of curve has many different solutions and the engineer has to decide whether a solution given by a program is practical or not. The engineer has to work hard and understand the curve and its geometry properly and vary his specifications, constraints and boundary conditions to get a few solutions before he gets an alternative which is easy to be implemented.
Note for Maintenance: Here, it is brought out that these constraints and boundary conditions are for the open line, engineers who work under the constraints of having to work only in limited traffic blocks and where OHE and other constraints are guiding factors. In the case of new constructions and gauge conversion works, the curves shall be laid to perfect geometry as designed and the realignment solution aiming at true circular curve shall only be followed, even if the same means extra efforts to improve badly laid/ adjusted curve during construction phase. If the curves are laid at the initial stage itself with compounding or averaging etc, these will give lot of trouble to the maintenance engineers and rectification of geometry will become very difficult once the trains start moving on the track.

The program for realignment of curves written by Shri M.S. Ekbote has various alternatives for realignment and gives good solutions for construction as well as open line. This program is available in the members’ area in IRICEN website http://www.iricen.gov.in. The latest version of the program is dated 31.03.2013. Detailed instructions for use of this program and basic logic are given at Annexure-I.

5.11 Attention to Curves in Electrified Sections: In electrified section, there is a limitation posed by the presence of the OHE masts and the track cannot normally be slewed in any direction more than 75-100 mm, or even less. Any realignment work in electrified section shall commence only after the electrical department has been informed in advance and it has confirmed the feasibility of the same. For larger slews, the electrical department has to adjust the OHE and/ or shift OHE masts. To tackle this problem, the realignment solution has to be found out where the slews are within limits at all stations. For longer curves or for very bad curves, it might not always be possible to adopt the realignment solutions worked out manually or through computer programs in electrified section due to the above restrictions. such situations shall be tackled as follows:

i. Partial improvement by limiting the maximum slews using computer programs available (The solution by averaging method or iterations method in the
Realignment program by Sh M S Ekbote or the method by iterations developed by Sh Venkateswara Rao).

ii. Improvement by compounding the curve can also be tried. The curve may be divided into a number of small segments and the solution for each segment can be obtained which is practical and meets the various site/ operational constraints. The segments will have uniform curvature and the curvature shall be in close proximity to the curvature of the stretches on either side. The solution by compounding in the realignment program developed by Mr M S Ekbote gives such a solution which can be used. This solution gives very good results in some curves which are disturbed badly in some part only and are OK in other parts.

iii. Part Curve Solution: The attending to part of curve is sometimes very beneficial as the complete curve is not bad and un-necessarily the complete curve shall not be attended due to this problem. For this, earlier the effort was required to be made manually only. However, for the first time, the program developed by Mr M S Ekbote gives an option for tackling only part of the curve. This option works well if the part is not very bad.

5.12 Getting better solutions using the software by Sh M S Ekbote:

The solution given by the computer program developed by Sh M S Ekbote can be improved to get reduced solutions by using the following methods:

- The minimum transition length is fixed by the speed and other considerations. However, we can always provide longer transitions. In the computer program, there is a facility of having unequal transition lengths on the either end of the curve. By changing the transition length, we can have some control over the slews and we can do iterations and choose from amongst the various options.

- Sometimes, there are locations where slews are excessive, and even after making all trials, the slews
are not coming within acceptable limits. Such a solution is not acceptable and feasible. However, we can manipulate the powers of computer program to give us an acceptable solution by inserting an imaginary obligatory point at the location and limit the slew to acceptable limit. By doing this, the curve gets distorted, but the result is feasible at site and sometimes may be the best feasible option.

5.13 Examples of Curve Realignment

Example 5.1: The versines have been surveyed for a curve as given in table below. Find out the realignment solution by string lining method.

<table>
<thead>
<tr>
<th>Stn</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
<th>16</th>
<th>17</th>
</tr>
</thead>
<tbody>
<tr>
<td>Versine</td>
<td>0</td>
<td>3</td>
<td>8</td>
<td>7</td>
<td>9</td>
<td>15</td>
<td>6</td>
<td>2</td>
<td>5</td>
<td>-4</td>
<td>15</td>
<td>16</td>
<td>18</td>
<td>20</td>
<td>8</td>
<td>7</td>
<td>3</td>
<td>0</td>
</tr>
</tbody>
</table>

Solution:

STEP I: Compute first and second summations for existing versines: Prepare summation Table – for existing versines, Ve as below. (It is desirable to make the table as below for error free calculations).

<table>
<thead>
<tr>
<th>Stn. No.</th>
<th>Ve</th>
<th>F.S. of Ve</th>
<th>S.S. of Ve</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>--</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>11</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>7</td>
<td>14</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>18</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>9</td>
<td>32</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>27</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>15</td>
<td>59</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>42</td>
<td></td>
</tr>
<tr>
<td>Stn. No.</td>
<td>Ve</td>
<td>F.S. of Ve</td>
<td>S.S. of Ve</td>
</tr>
<tr>
<td>----------</td>
<td>-----</td>
<td>-----------</td>
<td>-----------</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
<td>101</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>48</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>2</td>
<td>149</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>50</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>5</td>
<td>199</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>55</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>-4</td>
<td>254</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>51</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>15</td>
<td>305</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>66</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>16</td>
<td>371</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>82</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>18</td>
<td>453</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>100</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>20</td>
<td>553</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>120</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>8</td>
<td>673</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>128</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>7</td>
<td>801</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>135</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>3</td>
<td>936</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>138</td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>0</td>
<td>1074</td>
<td></td>
</tr>
</tbody>
</table>

**STEP II: Compute Beginning of curve, length of curve and proposed versines:** In the same summation table of step I, do further calculations for the beginning of curve, length of curve and proposed versines as shown below. The transition length for the curve is taken as 4 station units.
### FURTHER CALCULATIONS

**STEP a:** Calculate station/chainage of CC using eq (5.1)

\[ n - \frac{SSV_e \text{ upto } n}{FSV_e \text{ upto } n} \]

\[ \therefore \quad 17 - \frac{1074}{138} = 17 - 7.8 = 9.2 \]

**STEP b:** By Interpolation in the table, Offset at center of curve, \( O_c \) at station 9.2 = \( 2*(O_9 + 0.2*O_{10}) \)

\[ = 2*(254 + 0.2*51) = 528 \]

**STEP c:** Using eq (5.7),

\[ : \quad 528 = \left( \frac{138 \times 138}{4V^2} \right) \times V + \frac{V(L^2 - 4)}{12} \]

Taking \( L = 4 \) we get

\[ 528 = \frac{69 \times 69}{V} + V \]

Solving for \( V \) we get

\( V_p \) (i.e. proposed versine) = 9.2

**STEP d:** Using eq (5.6), The length of equivalent circular curve

\[ N = \frac{138}{9.2} = 15 \text{ stations} \]

\[ \therefore \text{ The total length of transitioned curve= 15 + 4 = 19 stations} \]

\[ \therefore \text{ Beginning of Curve = 9.2 - 19/2 = station no -0.3 and End of Curve = 9.2 + 19/2 = station no 18.7} \]

\[ \text{Slope of versine diagram in the transition portion } = 9.2/4 = 2.3 \]
STEP e: Calculations of proposed versines shall be done as follows:

<table>
<thead>
<tr>
<th>Station</th>
<th>Calculated versine</th>
<th>Rounded off to</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>—</td>
<td>0.0</td>
</tr>
<tr>
<td>0</td>
<td>0.3 x 2.3 = 0.69</td>
<td>0.7</td>
</tr>
<tr>
<td>1</td>
<td>0.69 + 2.3 = 2.99</td>
<td>3.0</td>
</tr>
<tr>
<td>2</td>
<td>2.99 + 2.3 = 5.29</td>
<td>5.3</td>
</tr>
<tr>
<td>3</td>
<td>5.29 + 2.3 = 7.59</td>
<td>7.6</td>
</tr>
<tr>
<td>19</td>
<td>—</td>
<td>0.0</td>
</tr>
<tr>
<td>18</td>
<td>0.7 x 2.3 = 1.61</td>
<td>1.6</td>
</tr>
<tr>
<td>17</td>
<td>1.61 + 2.3 = 3.91</td>
<td>3.9</td>
</tr>
<tr>
<td>16</td>
<td>3.91 + 2.3 = 6.21</td>
<td>6.2</td>
</tr>
<tr>
<td>15</td>
<td>6.21 + 2.3 = 8.51</td>
<td>8.5</td>
</tr>
<tr>
<td>Total versine over transitions:</td>
<td>36.8</td>
<td></td>
</tr>
</tbody>
</table>

Having known V<sub>p</sub>, EC, BC and versine slope, the versines on transition can be conveniently worked out by drawing a versine trapezium as shown below.

Fig 5.16: Versine Diagram for the proposed curve

STEP III: Calculate first and second summation of versine differences: Proposed versines shall be filled in table opposite existing versines. Computation of first summation, second summation of the versine difference shall be done as shown in step I. The final results are shown below:
<table>
<thead>
<tr>
<th>SN</th>
<th>Ve</th>
<th>Vp</th>
<th>Vp-Ve</th>
<th>F.S. of (Vp-Ve)</th>
<th>S.S. of (Vp-Ve)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>--</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0.7</td>
<td>0.7</td>
<td>0.7</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>3</td>
<td>0</td>
<td>0.7</td>
<td>+0.7</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
<td>5.3</td>
<td>-2.7</td>
<td>-2.0</td>
<td>+1.4</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
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<tr>
<td>Sum</td>
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</table>

**STEP IV: Application of correcting couple:** There is a residual second summation of +3. This very small value can be neglected as it is of no significance in field. However, for the sake of accuracy and explanation in this example, let us eliminate the same by using correcting couples.

Applying rules for applying correcting couple described in para 5.6.3.7 above, we require to put in correction with negative value in the initial stations and one with positive value in latter stations. Since correcting couple
shall be as small as possible, as far away as possible and equal and opposite, let us put a small correcting couple of -0.1 mm at station no 0 and corresponding +0.1 mm at station no 18. (farthest stations in the curve)

By mental arithmetic, we can see that the second summation of this correcting couple at last station will be -1.8 mm (This can be calculated by value of the correction* distance between the two corrections, i.e. -0.1 mm * 18).

This is less than 3.0 mm, so we require another correcting couple, with second summation 3.0 mm – 1.8 mm=1.2 mm. Let this be also of value 0.1 mm, and provided between stations 3 and 15 (The second summation of this correcting couple is -0.1 mm * 12=-1.2 mm)

Proceeding further from the computations shown in step III, the computations after the correcting couples are applied are as below:

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<td></td>
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<td>-1.7</td>
<td>+16.1</td>
<td>+32.2</td>
<td>+32</td>
<td>9.2</td>
</tr>
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<td>16</td>
<td>9.2</td>
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<td>+30.9</td>
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<td>+62</td>
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<td>+41</td>
<td></td>
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<td>-2.1</td>
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<td>+77.8</td>
<td>+78</td>
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<td>+29</td>
<td></td>
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<td>-2.5</td>
<td>+26.5</td>
<td>+53.0</td>
<td>+53</td>
<td>9.2</td>
</tr>
</tbody>
</table>
Example 5.2: If the station 12 in the curve data given in example 5.1 happens to be obligatory and no slew is permitted, how will the solution change?
Solution:

The station 12 had the maximum slew of +78 mm as seen in the solution above. If no slew is permitted at this location, the procedure for finding out the solution will be as follows:

**STEP I: Equate the offset of existing curve with that of proposed curve at the station no 12:** The example 5.1 has been done by equating the offset from the tangent at the center of curve for the existing curve and the proposed curve. Here, since there is restriction of slew at station no 12, existing offset at Station 12 shall be equated to the offset at station 12 due to proposed curve. As discussed earlier, the offset for the proposed curve shall be in two parts, offset for equivalent circular curve and the shift due to proposed transition.

It may be noted that now the designed versine will be altered in such a way that the proposed transitioned curve passes through station 12 of existing curve and due to the same, the slews will increase. Therefore, it is seen that the objective of passing the proposed curve through a specific
point of the existing curve can be achieved at the cost of increased maximum slew.

As per computation done in the example 5.1, Offset of the existing curve at Station 12, i.e. \( O_{12} = 2 \times \) second summation of the existing versines upto station no 12 = 2 x 453 = 906.

Now, Designed offset at station 12 due to proposed circular curve = \( VT^2 \) where \( T \) is the tangent length upto station no 12, which is taken equal to the length of circular curve due to the large radius of the railway curves. Using eqn. (5.7),

\[
T = \frac{N}{2} + (12 - CC) \quad \text{where} \quad N = \frac{\text{Total F.S. of } V_c}{V}
\]

Designed offset due to shift = \( V \times \frac{(L^2 - 4)}{12} \) = \( V \), since \( L = 4 \) as earlier

\[
906 = \left( \frac{138}{2V} + (12 - 9.2) \right)^2 \times V + V
\]

Solving this, we get \( V = 11.08 \)

\[
N = \frac{138}{11.08} = 12.46 \quad \text{and} \quad N' = N + L = 16.46
\]

And, Versine slope = \( \frac{11.08}{4} = 2.77 \)

Using equation (5.9), Beginning of curve,

\[
BC = 9.2 - \frac{16.46}{2} = 0.97
\]

Using equation (5.10), End of curve,

\[
EC = 9.2 + \frac{16.46}{2} = 17.43
\]

Versine diagram and design of versines on transition for the proposed curve.
Table showing calculations of proposed versines.

<table>
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<tr>
<th>Station</th>
<th>Calculated versine</th>
<th>Rounded off to</th>
</tr>
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<tbody>
<tr>
<td>0</td>
<td>—</td>
<td>0.0</td>
</tr>
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<td>1</td>
<td>$= \frac{0.03 \times 11.08}{4} = 0.08$</td>
<td>0.1</td>
</tr>
<tr>
<td>2</td>
<td>$= 0.8 + 2.77 = 2.85$</td>
<td>2.8</td>
</tr>
<tr>
<td>3</td>
<td>$= 2.85 + 2.77 = 5.62$</td>
<td>5.6</td>
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<td>$= 5.62 + 2.77 = 8.39$</td>
<td>8.4</td>
</tr>
<tr>
<td>18</td>
<td>—</td>
<td>0.0</td>
</tr>
<tr>
<td>17</td>
<td>$= \frac{0.43 \times 11.08}{4} = 1.19$</td>
<td>1.2</td>
</tr>
<tr>
<td>16</td>
<td>$= 1.19 + 2.77 = 3.96$</td>
<td>4.0</td>
</tr>
<tr>
<td>15</td>
<td>$= 3.96 + 2.77 = 6.73$</td>
<td>6.7</td>
</tr>
<tr>
<td>14</td>
<td>$= 6.73 + 2.77 = 9.5$</td>
<td>9.5</td>
</tr>
<tr>
<td></td>
<td>Total:</td>
<td>38.3</td>
</tr>
</tbody>
</table>

Taking $V_p$ as 11.1 instead of 11.08 the total versines on circular position will be 99.9. Therefore, to adjust the total to 138-38.3= 99.7 mm, versine at stations 5 and 13 shall be taken as 11. The result can be seen in the following table:
Table showing results with an obligatory point at station 12

| SN | Ve | Vp | S.S. of Vp-Ve | Vc | S.S. of Vc | Final Vp | Final slews rounded off to nearest mm = 
| 2 x (SS(Vp -Ve)+SSVc) |
|---|---|---|-------------|---|-------------|---------|----------------------------------|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 3 | 0.1 | 0 | 0.1 | 0.1 |
| 2 | 8 | 2.8 | -2.9 | 2.8 | -6 |
| 3 | 7 | 5.6 | -11 | 5.6 | -22 |
| 4 | 9 | 8.4 | -20.5 | 8.4 | -41 |
| 5 | 15 | 11 | -30.6 | 11 | -61 |
| 6 | 6 | 11.1 | -44.7 | -0.1 | 11.0 | -89 |
| 7 | 2 | 11.1 | -53.7 | -0.1 | 11.1 | -107 |
| 8 | 5 | 11.1 | -53.6 | -0.2 | 11.1 | -107 |
| 9 | -4 | 11.1 | -47.4 | -0.3 | 11.1 | -95 |
| 10 | 15 | 11.1 | -26.1 | -0.4 | 11.1 | -53 |
| 11 | 16 | 11.1 | -8.7 | -0.5 | 11.1 | -18 |
| 12 | 18 | 11.1 | +3.8 | -0.6 | 11.1 | +6 |
| 13 | 20 | 11 | +9.4 | +0.1 | -0.7 | 11.1 | +17 |
| 14 | 8 | 9.5 | +6.0 | -0.7 | 9.5 | +11 |
| 15 | 17 | 6.7 | +4.1 | -0.7 | 6.7 | +7 |
| 16 | 3 | 4 | +1.9 | -0.7 | 4 | +2 |
| 17 | 0 | 1.2 | +0.7 | -0.7 | 1.2 | 0 |
| 18 | 0 | 0 | +0.7 | 0 | 0 | 0 |

Notes:

1. Slew at station 12 is only + 6mm as against 0 required due to rounding off errors.
2. The max. slew is –107 mm at stations 7 & 8
3. The designed versine is +11.1 as against 9.2 earlier. This means that the curve has got sharper than earlier due to the obligatory point restriction.
4. The error of +6 in SS of (Vp-Vc) can be safely ignored, but it is eliminated for the sake of accuracy, by application of a correcting couple without disturbing the versines very much.
Example 5.3:
Versines recorded in portion of a curve is shown in table below. What action is required to be taken? Find out the solution to the problematic stretch.

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<th>21</th>
<th>22</th>
<th>23</th>
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<tbody>
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<td>32</td>
<td>29</td>
<td>30</td>
<td>26</td>
<td>30</td>
<td>28</td>
<td>40</td>
<td>32</td>
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</tr>
</tbody>
</table>

Solution:
On scrutiny of the existing versines, we see that the complete curve is not bad. Only a portion of the curve is having station to station versine difference. This is a problem which shall be tackled by local adjustments and not complete realignment. The variation is seen between stations 24 to 36. Here, the choice of the stations between which the curve is considered disturbed is by engineering judgment and we can consider the curve to be disturbed from station no 23 or 25 without affecting the result of curve much. However, longer the curve chosen for adjustment more will be the length of track to be attended. As per principles of realignment, we take a few stations on either side of affected portion and find out a solution for local adjustment of curve at stations 23 to 38 as below.

STEP I: The sum total of the versines in station 23 to 38 is 480. There are total 16 stations, so the proposed versine shall be 30. Using the proposed versines, let us find out the versine difference, first summation and second summations for stations 23 to 38.

STEP II: The second summation of the versine difference at the station no 38 comes to +12, so correction of -1 mm is applied at station no 25 and corresponding correction of +1 mm at station no 37 (so that the second summation of the correcting couple is -1 mm * 12 = -12 mm, as required). Second summation of the correcting couple is found out.

STEP III: The second summation of versine difference in Step I and the second summation of correcting couple in Step II shall be added up to get the final second summation. Twice this value gives the slews at each of the stations.
Computations for local adjustment of curve as per above procedure are done as follows:

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<th>Stn No</th>
<th>Ve</th>
<th>Vp</th>
<th>VP-Ve</th>
<th>F.S. Vp-Ve</th>
<th>S.S. Vp-Ve</th>
<th>Vc</th>
<th>F.S. Vc</th>
<th>S.S. Vc</th>
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<th>Final Versine</th>
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Example 5.4:
While solving the example 5.1, the transition length was assumed to be 4. If the speed on the section is proposed to be increased, the bottle neck is the transition length and the same is to be increased to 6 station units. Find out the slews required for this situation.

Solution:

Now, if we provide the longer transition, the shift will also increase. The shift increases by square of transition length and the same will increase by 2.25 times in this case where the length of transition is 1.5 times the original length. The entire curve will have to be shifted inwards if we follow this. However, we know that the curve shifts outwards if the radius is reduced, so a method of reducing the amount of slews is that the circular curve be made sharper (radius may be reduced). Therefore, for minimum slews, the existing offset at CC is equated to offset at CC due to proposed circular curve plus the offset due to the revised shift.

From example 5.1, we already know that for the existing curve, CC = 9.2 and

Offset at center of curve, O_c at 9.2 = 528

Equating this to proposed shift at CC, using eqn. (5.7), we get

\[ 528 = \frac{138 \times 138}{4V} + \frac{V(36-4)}{12} \]

\[ = \frac{69 \times 69}{V} + \frac{8V}{3} \]

Solving this quadratic equation for V, we get V = 9.76, say 9.8. (Since the versine in circular portion, 9.8 is more than 9.2 in example 5.1, it is clear that the radius has been reduced)

Length of equivalent circular curve i.e.

\[ N = \frac{138}{9.76} = 14.14, \quad N' = (N + L) = 14.14 + 6 = 20.14 \] stations
\[ BC = 9.2 - \frac{20.14}{2} = -0.87 \]

\[ EC = 9.2 + \frac{20.14}{2} = 19.27 \]

Versine diagram and design of versines for the proposed curve are as follows:

**Fig. 5.17: Versine Diagram for the proposed curve**

Versines on transitions i.e. at stations 0 to 5 and 14 to 19 can be worked out in a manner similar to the one shown in example 5.1. The versines at station nos – 1 and 20 are zero. The results of computed slews are shown below:
Table showing the results for increased transition length.

<table>
<thead>
<tr>
<th>S No</th>
<th>Ve</th>
<th>Vp</th>
<th>S. S. of Vp-Ve</th>
<th>Vc</th>
<th>S. S. of Vc</th>
<th>Final Vp</th>
<th>Final slew rounded off to nearest mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0</td>
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</table>

Notes:  1) Even though the transition length is increased by 1.5 times, the maximum slew is only +70.

2) The slew at station 9.2 is close to zero. (The slew at Station 9 being -25 and at Station 10 being +17)

3) The SS of versine difference of +2.9 at last station is eliminated by application of small correcting couple, for the sake of accuracy of calculations. (Otherwise this can be left
as it is without affecting the results in any manner)

4) The versine in the central portion of the curve in example 5.1 was 9.2, which has now changed to 9.8, indicating increase in degree or sharpness of curve.

**Example 5.5:** A curve has the following readings at one end:

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<td>43</td>
<td>26</td>
<td>25</td>
<td>31</td>
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</table>

Find out the solution for rectifying the versines.

**Solution:** Proceeding as earlier, computations are done as below.

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<th>S.S. of Ve</th>
</tr>
</thead>
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</tr>
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</tbody>
</table>

Transition length = 80m i.e. (8 stations)

Portion to be tackled from station N0.-2 to station No.11

Offset at station no 11, \( O_{11} = 2 \times 1166 = 2332 \)

From Equation (5.8)

\[
2332 = V \left[ \frac{240 + \frac{V}{2}}{V} \right]^2 + \frac{V(L^2 - 4)}{12}
\]

\[
V^2 - 398.48V + 10971.43 = 0
\]

Solving for \( V \), we get

\( V = 29.76 \) say 29.8 mm

Versine slope: \( \frac{29.8}{8} = 3.7 \) mm

\[
BC = 11 - \frac{240 + 14.9}{29.8} - 4
\]

= -1.554

Versine at Station – 1 = 0.554 x 3.7

= 2.064, Say 2.1
Now, the solution can be worked out as follows:

<table>
<thead>
<tr>
<th>Stn. No.</th>
<th>Ve</th>
<th>Vp</th>
<th>Vp-Ve</th>
<th>F.S. Vp-Ve</th>
<th>S.S. Vp-Ve</th>
<th>Full slew</th>
<th>Final Vp</th>
</tr>
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Note: a) 0.1 is added to versine at stations 7 to 10 to adjust the total of proposed versine as 240.

b) Residual slew of +2 mm at station 10 is neglected and is adjusted locally by giving a slew of + 1 at station 11.
5.14 CHAPTER V REVISION QUESTIONS

1. What is the criteria for realignment of a curve?

2. What is an obligatory point in reference to curve realignment?

3. Amongst the various options for realignment viz, true circular curve, compounding and averaging in the computer program developed by Sh. M. S. Ekbote, which one is best suited to open line and which one to construction works? Why?

4. Explain the procedure to be adopted while carrying out realignment in field by tamping where slews are excessive (more than 75 mm).

5. True or false:
   a. Gauging shall be done after the realignment is done
   b. If a curve is to be realigned, the main objective is to revert to original alignment.
   c. Curve realignment is a problem about choosing the best amongst the various options available.
   d. If a curve is to be realigned, curve length shall not be changed
   e. When choosing the proposed versines, the sum of existing versines shall be equal to the sum of proposed versines.
   f. The slews at the center of curve in any realignment solution shall be zero.
   g. Correcting couple shall be as small as possible.
ANNEXURE I

A Write up on the Computer Programs for Realignment of curves Developed by Shri M.S. Ekbote, Addl. Member (CE) (Retired) (Revised on 31-03-2013)

1.0 Introduction - The Subject of “Realignment of existing curves" has been a matter of considerable interest among Railway Permanent Way Engineers. It has been the dream of every track Engineer to give a perfectly smooth and well transitioned curve to give absolutely smooth riding on the curves. Over last 50-60 years a number of theories have been developed for realigning existing curves by various engineers with considerable success. Indeed it is one of the most fascinating subject on which the interest will ever continue.

This program was initially put in the Member's downloadable area of IRICEN's website on 18-12-2006. Thereafter the program has been presented to some batches of Trainee officers undergoing various training programs at IRICEN and following feed back was given to further improve the program.-

- The users should not be required to remember and enter the file name with path etc but should be able to select the file from the normal file open dialogue box as is the case in all window based programs. Likewise output file name and path should also be selected using a file save dialogue box.
- The user should not be required to remember number of stations as any error in writing this figure results into a runtime error.
- The user should be able to create a data file through the program itself rather than creating a file independently in the notepad.
- The program should detect and warn the user of any
missing entries i.e. inadvertently left over entries or omission in completing data in text boxes

All these modifications were made in earlier versions (last revision of 31-08-2007) and in this revision following further improvements have been made-

- The program also caters for correction to part of the circular curve where full curve does not need realignment.
- For compounding option at junction of the two adjacent parts of curve the program has been modified to cater to 6 station transition if the versines of the two parts differ greatly. Earlier there was a provision of only two station transition.
- Some further changes have been made to correcting couple subroutine and now invariably the residual slew at the last station should become zero which in earlier version used to be up to 2 or 3 mms in some cases due to rounding off errors.
- On clicking the Reset button the data entered for obligatory points does not get erased but if you change the number of obligatory points to zero then the obligatory point data gets erased fully.

2.0 Brief Background - Whatever is the algorithm to determine the proposed Versines there are only two basic conditions which any solution should satisfy namely-

(a) The sum of existing and proposed Versines (1st Summation) should be equal.

This ensures that that there is no change in deflection angle, and

(b) The C.G. of the Versine diagram of the proposed and the existing Curve should lie exactly at the same point. This is achieved by equating the 2nd summation of Versines of the existing and proposed curves and suitably selecting the Starting station. This ensures that there is no residual slew (shift) of the curve at the end. With these two basic conditions any algorithm can be conceived by a User/programmer and a
solution can be found. In practice matching of C.G. of the existing and proposed curves is done to get solution and the same is fine-tuned by applying correcting couples.

3.0 Computer Program - In this note we will not cover the theoretical derivations of the Algorithms used but only a brief mention of the algorithm will be made. The computer Program is highly user friendly. This program consisted of a number of individual Programs written originally in BASIC but were later amalgamated in one consolidated Program in Visual Basic, which is the successor to old DOS based BASIC. The Programs have a very attractive user interface.

4.0 Instructions for installation of the Computer Program

After downloading the Zip folder from the IRICEN’s website unzip the zip folder and extract to a folder of your choice. Open the realign_curve folder and you will see a setup file with computer icon. Just double click it, follow the instructions and the program will be installed on your computer. It will install all relevant files to run the package. You can run the program by going through start menu, point to programs and to realign_curve and program will automatically get loaded and you can run it in a normal way. It is suggested that users may install the program on their computers as this will not cause any problem.

(Alternately the computer Program realign_curve.exe can be Run directly using the run command in the start menu. Click Start go to run menu Browse and locate the folder where the program is saved in your computer and click OK the program will get loaded.

In this method the program requires two active X files available and properly registered. These Files are MSFLXGRD.OCX and MSCHRT20.OCX. There is a possibility the user may get a run time error “…. OCX” file not found or properly registered then user should copy these files in the System32 directory of Windows folder in the C drive. These files have been included with this write up. For registering these files open DOS command C:\- prompt by going through Start and accessories then change directory windows/system32 directory and write Command regsvr32
MSCHRT20.OCX and press <enter> you will get a message of Successfully registering the .OCX file. Likewise other .OCX file can also be copied and Registered. On completion of registration of these files (Necessary only where the runtime Error is encountered) you would be able to run the program successfully.

On loading the program the following opening screen should be visible.

As can be seen there are 4 option buttons available for selection. Let us select the 1st Choice of simple curve and click proceed you will see the following screen.
4.1.1 Simple curve - This Choice has 5 options i.e. True circular curve, compounding by dividing curve in segments (Not more than 5), Solution by averaging method, Solution of a part curve and a novel approach to restrict the maximum slew at the user's choice. The P. Way engineer desiring to use this program for realignment has to decide the method to be used. The same is to be used based on the degree of disturbance in the curve and the degree of correction desired. The three methods will give results suitable for different situations. In case the engineer is not clear as to the method which will give the most suitable result, the program may be run in different options and the most suitable result output may be selected for actual use. It may be noted that only white text boxes require a data entry by the user. All other text boxes get their entries through the program.

All the choices require a data file. The data file to be used is a simple collection of numbers i.e. the versine values from beginning to end of the curve. The file can be conveniently created in Notepad and saved in any folder. Alternately the data file can be created by entering the “NO” choice after you have entered the station Number of first station and clicked the NEXT button. The station number at the Beginning is to be given by the user after carefully perusing the Existing Versines. If you want to use an existing Data file then click “: YES” button in the message box. The data to be entered is all in station units. The sample data file given with this write-up is C56.txt and has been given for the user to try. For entering the input data file please click the Command button “Browse for input file...” and file open dialogue box will open for to selecting the input Data file. After clicking the NEXT Button the following Dialogue Box is displayed.
Assuming that you have clicked the “NO” button then following Data entry screen is displayed.

The station number of first station is automatically picked up from earlier screen and on entering the versine data click the ‘Enter’ button or alternately press the Enter Key on the keyboard and the data get transferred to the grid shown there. The next station appears for data entry in the textboxes. You can go on entering the versine values and press the Enter key on the keyboard (or click the enter button) and the values will appear in the Grid shown there. When you reach the last station for which data is to be entered, click the ‘End’ button. The ‘Edit’ and ‘Over’ buttons will get activated. Using these buttons you can modify any incorrect entries made. Enter station number and the versine value as entered earlier automatically appears in the versine text box. You can then correct the versine value at the station and then press ‘Edit’ so that the corrected value gets replaced in the grid. When all the corrections have been made and you are satisfied
regarding the data input, click ‘Over’. Press ‘Save and exit’ button when you are done with all modifications and the computer will prompt for the name and location where the file is to be saved. The file is saved in Notepad and the same can be accessed if the program is to be run again with the same Data and there is no need for entering the data again and again.

In case the data is already typed in notepad or previously created as a notepad file by running the program, you shall select ‘YES’ button for using an existing Data file then Click ‘browse for input file’ button to locate the file. After locating and selecting input data file and completing the data on transition type and length click the ‘Compute’ button. The following screen is displayed:

[Image of the screen showing data entry and computation]

The “show result” and “show chart” buttons become enabled. The page gives sufficient details about the solution.

In certain cases the program may ask the user to add a few stations in the beginning of curve or end of the curve with zero versines. This may happen when the bulk of the existing versines are grouped together resulting in the calculated length of the curve being bigger than the data
range for which versine data has been fed. To understand this point it may be advisable to know a bit about the logic of the program which is as under:

After calculating first and second summation, the program calculates the station number of the CG of the curve. Even if you add a few stations with zero versines at the beginning or end numbering -1, -2, -3 and so on, the station number of CG of the curve would remain unchanged.

Then it calculates the offset from the first tangent at the CG of the curve.

Then it calculates versine (Radius) of an ideal curve which has the same off set at the CG of the Curve. For this a quadratic equation is to be solved. The mathematical formulation of this equation is available in our paper published in the P way bulletin of October 1986.

Then it calculates length of the curve based on the calculated Versine and Sum of existing Versines. Then first and last station no of the proposed curve is calculated by subtracting and adding half the curve length from the CG of the curve. It would thus be clear that this recalculated beginning and end of the curve should be within the outer most data points for which Versine (could be zero) values are fed.

Then it assigns the proposed versines along the station numbers to the curve. Checking the difference between Sum of proposed and existing versines the difference (which will be very small) is distributed uniformly along the curve to ensure that sum of proposed and existing versines is equal.

Then it calculates slews and finds out the value of max positive and maximum negative slews. If the algebric difference between them is more than 10 mms then a correction is applied to Offset at the centre of curve and procedure from step no 3 is repeated till the algebric difference does not become less than or equal to 10 mms. This is done to ensure an overall reduction of slews along the curve.

Correcting couple is then applied based on residual slew at the last station of the curve and proposed versines are recalculated.
The Slews are rounded off to nearest whole number and proposed Versines are calculated back so as to ensure mathematical compatibility of the solution.

When the user encounters the message to add stations in the beginning or at the end of the curve the user should open the input file in notepad add the number of stations as instructed and re-run the program after revising starting station number. After you have run the program and got the screen as previously obtained clicking the show chart button gives a graphical appreciation as below.

The Show result button will list the solution for ready reference. The Show result and Show Chart buttons work like toggle switches. We have additional features such as-

1) Choosing of a maximum of 5 obligatory points for restricting slews.

2) For the choice of true circular curve we can have either Cubic parabola, ‘S’ Shaped (4th order parabola), or Sine transitions.

3) It is also possible to select different transition lengths for the two ends of the curve.

4) It is however mentioned that different transition length case is only possible with cubic Parabola transition.
After the user is fully satisfied about the solution he can print the page for record to keep the data entered in getting the solution and copy the results to a file by clicking the 'copy to a file' buttons. The file is saved in notepad and shall be suitably named in the dialogue box opened. Upon saving, the program prompts whether the tamping data is required. If the same is required, the sleeper density is asked by the computer and the slew to be marked on the alternate sleepers comes which can be saved as separate notepad file.

4.1.2 We now discuss the second option of finding a solution by compounding. Where we find excessive slews by 1st method this is an ideal approach. In this method the user has to select the station numbers for end of each segment by trial and error. As a rough guide it may be advisable to keep segment lengths of roughly 15-20 station units. We now solve the same curve using this method and for unequal transition lengths. The screens appear like this-
Output of this problem is as below:
The Slews have been drastically reduced and curve is still quite smooth. We can further reduce the slews by restricting slews at obligatory points.

The method attempts at equating second summation of the existing and proposed curves at the end of each segment and the residual of the 1st summation is thrown in the last segment, which is treated from the trailing end.

4.1.3 The third option is by adopting the averaging method. This is not a scientific approach but will be useful where the existing curve is badly distorted. In this method proposed versines are taken as average versine of 3 stations and successive iterations are done at user’s choice. The Screen after a solution would look like as below-

![Screen capture of a software interface for curve adjustment with field data input and output options.]

The choice to reduce slews is still available by restricting slews at obligatory points.

4.1.4 The 4th option pertains to solving a curve for part solution. The earlier steps remain as same but on clicking option button for this option two text boxes become visible for entering the station numbers between which the realignment is sought. The file used for this solution is Cpart included with this write up and is based on the illustration given in the
IRICEN booklet “Railway curves” After repeating the earlier steps, selecting the appropriate file and clicking option 4 we get the following screen. Here we can enter the station numbers between which the correction is required. We also see a “Show Ex Versine” button clicking which we can see the input file and confirm our station selection. The display of existing versines gets closed when we either click hide button or click compute button. The final result appears as shown in the following screen-

In case the residual Slew between selected points becomes too excessive and can not be adjusted with an acceptable correcting curve a suitable message is displayed. On clicking OK button on the message box the screen gets cleared and user has option to change selection of stations.

4.1.5 The 5th method is to restrict the slews to a predetermined value. This method is too simplistic and will not result in uniform versines. This method should be adopted only when it is not possible to find a satisfactory solution by any other method and site has too severe difficulties in field and slews have to be restricted at any cost. The solution of same curve with restricting slews to say 125 mms is as under
4.2 Reverse Curves- The second option in the 1st screen is that of reverse curves. Here the screens and results are of similar type. The sample data file we use will be rev4.txt (given with the write up). It has 50 stations. In case of reverse curves also, data file can be created interactively as explained in the case of simple curve but the starting station is to be carefully selected. There are two options i.e. one where Junction station is known and second where two parts of the curve are almost of equal curvature. The 1st case refers to cases where the leading and trailing part of the reverse curves have widely varying versines and are generally in midsections where as the second case refers to curves laid in approaches of Bridges or yards as in case of doublings to get requisite track centers at these locations. The output with this sample curve is as shown -
In both cases of simple and proposed curves we can copy results to a file and take a print out. After you click the "Copy to a button" and it will open file save dialogue box the
default name is test.txt which can be changed by you. In addition the slewing data for tamping machines can also be copied to another file through a similar dialogue box.

4.3 Realigning the Transitions- The third choice in the 1st screen is of realigning the transitions alone. Here all data is to be given on the screen and the data entry and output screens look like this-

Output screen is as below-
4.4 Vertical Curves- The last Choice is that of Vertical Curves. It is strictly not connected with the problem of realignment but has been included with the intention of making the program self-contained. The opening Screen and the sample outputs are as below-

The results after clicking compute button are as below-

Conclusions- All the aspects of using the computer program can not be fully covered in a note, it is however felt that users by trial and error can comfortably feel at ease and use the program. It is felt that the program can be of great use to field Engineers.